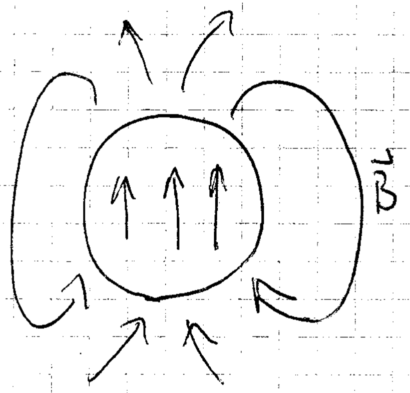
Naloga 1

Velja $\vec{B} = \mu_0(\vec{H} + \vec{M})$, ker je $\vec{M}$ homogen, pa rezen na površini oba izpolnjujeta Laplaceovo enačbo.



Zunanje polje je polje dipola,

$$\vec{B}^z = \frac{\mu_0}{4\pi} \frac{3(\vec{m}\cdot\vec{r})\vec{r} - \vec{m} r^2}{r^5}$$

$$\vec{m} = V \cdot \vec{M} = \frac{4\pi R^3}{3} \cdot \vec{M}$$

Notranje polje mora izpolnjevati robne pogoje

$$B_r^n\Big|_R = B_r^z\Big|_R \qquad \text{in} \qquad H_\vartheta^n\Big|_R = H_\vartheta^z\Big|_R$$

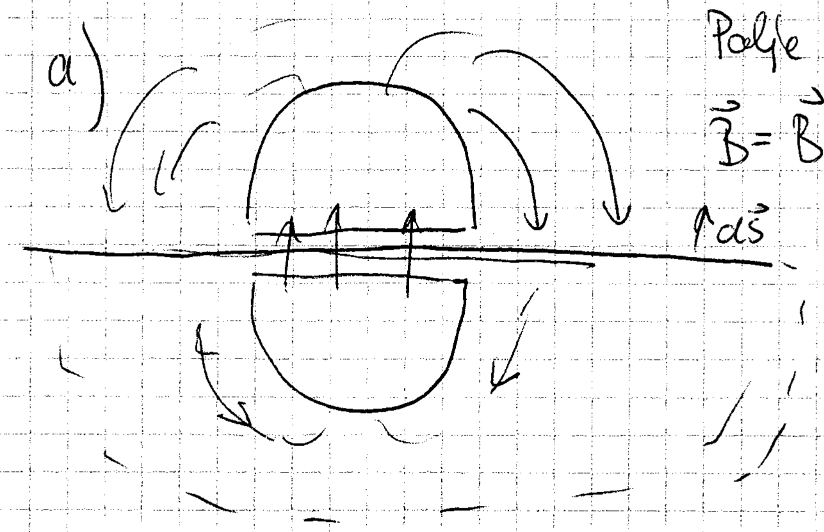
$$B_r^z\Big|_R = \frac{\mu_0}{4\pi} \frac{2(\vec{m}\cdot\hat{e}_r)}{R^3} = \underline{\frac{2}{3}\mu_0 \vec{M}\cdot\hat{e}_r}$$

$$H_\vartheta^z\Big|_R = \frac{1}{4\pi} \frac{+m \sin\vartheta}{R^3} = \frac{1}{3} M \cdot \sin\vartheta$$

Te r.p. izpolnjuje homogeno polje

$$\vec{B}^n = \frac{2}{3}\mu_0 \vec{M} \qquad \text{in} \qquad \vec{H}^n = -\frac{1}{3}\vec{M}$$

a)



Polje v špranji:

$$\vec{B} = \vec{B}^{M} = \underline{\underline{\frac{2}{3}\mu_0 \vec{M}}}$$

$$F_z = \oint T_{zz}\, dS \qquad T_{zz} = \frac{1}{\mu_0}\left(B_z^2 - \frac{1}{2}B^2\right) \overset{B=B_z}{=} \frac{B^2}{2\mu_0}$$

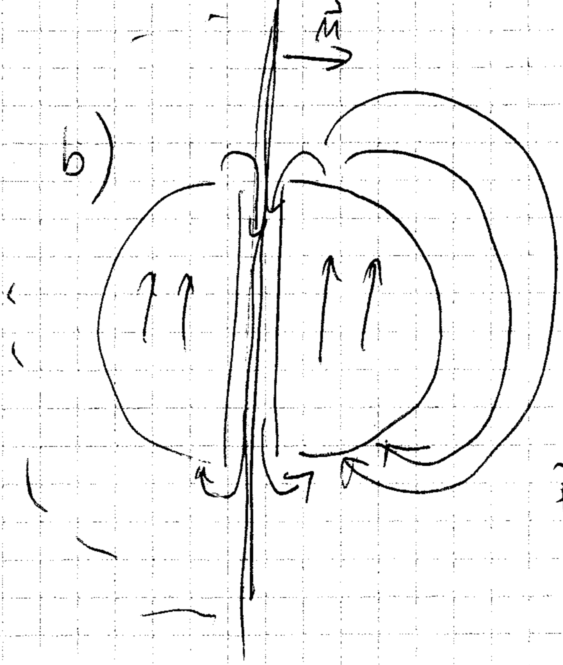
$$F_z = \frac{1}{2\mu_0}\int B^2 dS = \frac{1}{2\mu_0}\left(\pi R^2 \cdot B_M^2 + \int_R^{\infty} 2\pi r\, dr\, B_z^2(r)\right)$$

$$= \frac{1}{2\mu_0}\left(\pi R^2 \cdot \left(\frac{2}{3}\right)^2 \mu_0^2 M^2 + \int_R^{\infty} 2\pi r\, dr \cdot \frac{\mu_0^2}{(4\pi)^2}\left(\frac{4\pi R^3}{3}\right)^2 \frac{\vec{M}^2}{r^6}\right)$$

$$= \frac{2}{9}\mu_0 \cdot \pi R^2 M^2 + \frac{1}{9 \cdot 4}\mu_0 \pi R^2 M^2 = \underline{\underline{\frac{1}{4}\mu_0 \pi R^2 M^2}}$$

Ker na spodnjo polovico deluje sila v pozitivni z-smeri, je sila <u>privlačna</u>.

b)



$$F_x = \oint T_{xx}\, dS = \frac{1}{\mu_0}\int \left(B_x^2 - \frac{1}{2}B^2\right) dS$$

$$\overset{B_x=0}{=} -\frac{1}{2\mu_0}\int B^2\, dS$$

Polje v špranji: $\vec B_{šp.} = \mu_0 \vec H^M = \underline{\underline{-\frac{1}{3}\mu_0 \vec M}}$

$$F_x = -\frac{1}{2\mu_0}\left(\pi R^2\left(-\frac{1}{3}\mu_0\vec M\right)^2 + \int_R^\infty r\,dr \int_0^{2\pi} d\vartheta\, \frac{\mu_0^2}{(4\pi)^2}\cdot \frac{9(\vec m\vec r)^2 r^2 + (\vec m^2) r^4 - 6(\vec m\vec r)^2 r^2}{(r^5)^2}\right)$$

$$= -\frac{1}{2\mu_0}\left(\frac{\pi}{9}R^2\mu_0^2 M^2 + \int_R^\infty r\,dr \int_0^{2\pi} d\vartheta\, \frac{\mu_0^2}{(4\pi)^2}\frac{3\cos^2\vartheta+1}{r^6}\left(\frac{4\pi R^3}{3}\right)^2 M^2\right)$$

$$= -\frac{1}{2\mu_0}\left(\mu_0^2\,\frac{1}{9}\pi R^2 M^2 + \mu_0^2\cdot\frac{1}{9}\,5\pi\cdot\frac{1}{4}R^2M^2\right)$$

$$= \underline{-\frac{1}{8}\mu_0\pi R^2 M^2}$$

Sila je ~~[illegible]~~ odbojna.

Naloga 2 in 3 sta skoraj identični nalogam s prejšnjih izpitov.