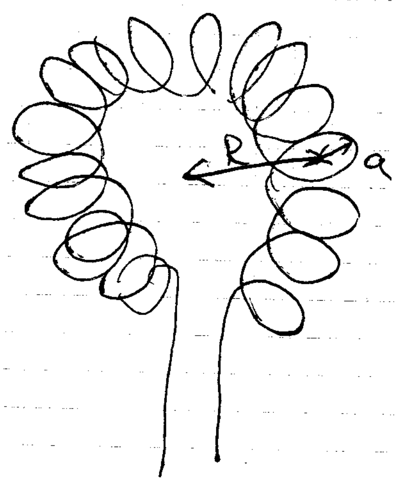
1)



a) Zaradi simetrije ima polje $\vec{B}$ lahko samo smer $\hat{e}_\varphi$.

Po Amperovem izreku velja

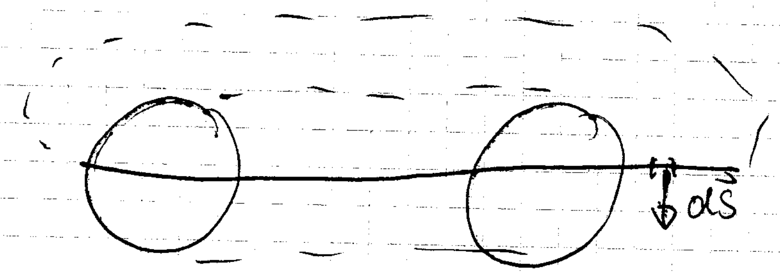
$$\oint \vec{B}\,d\vec{l} = \mu_0 \cdot \sum_i I_i$$

$$\Downarrow$$

$$B \cdot 2\pi R = \mu_0 \cdot N \cdot I$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi R}}$$

b)



Sila na zgornji del:

$$\vec{F}_z = \int \overleftrightarrow{T} \cdot d\vec{S} = -\int \overleftrightarrow{T} \cdot \hat{e}_z \, dS$$

$$F_z = -\int T_{zz}\, dS = -\int \frac{1}{\mu_0}\Big(\underbrace{B_z B_z}_{=0} - \frac{1}{2} B^2\Big) dS =$$

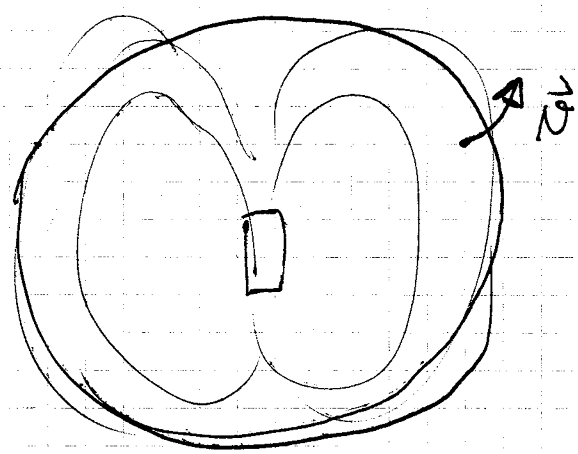
$$= \frac{1}{2\mu_0} \cdot B^2 \cdot \underset{2\pi R \cdot 2a}{\underline{S}} = \frac{\mu_0 N^2 I^2 \cdot a}{2\pi R}$$

Žica prečka to ravnino $2N$ - krat.

Zato je $$F_{žice} = \frac{F_z}{2N} = \boxed{\frac{\mu_0 N I^2 a}{4\pi R}}$$

2) Polje magnetnega dipola:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{3(\vec{m}\vec{r})\vec{r} - \vec{m}\, r^2}{r^5}$$



Sila na nosilce naboja: $\vec{F} = e(\vec{v} \times \vec{B} + \vec{E}) =$

$$= e\left((\vec{\omega} \times \vec{r}) \times \vec{B} + \vec{E} \right) =$$

$$= e\left(\vec{r}(\vec{\omega}\vec{B}) - \vec{\omega}(\vec{r}\vec{B}) + \vec{E} \right)$$

Tangencialna komponenta te sile mora biti 0,

$F_\vartheta = 0$

$$\underbrace{(\hat{e}_\vartheta \cdot \vec{r})}_{=0}(\vec{\omega}\vec{B}) - \underbrace{(\hat{e}_\vartheta \cdot \vec{\omega})}_{-\omega \cdot \sin\vartheta}(\vec{r}\vec{B}) + E_\vartheta = 0$$

$$E_\vartheta = -\omega \cdot \sin\vartheta \cdot (\vec{r} \cdot \vec{B}) = -\omega \cdot \sin\vartheta \cdot \frac{\mu_0}{4\pi} \frac{3(\vec{m}\cdot\vec{r})\cdot r^2 - \vec{m}\vec{r}\cdot r^2}{r^5}$$

$$= -\frac{\mu_0 \cdot \omega \cdot m}{4\pi R^2} \cdot 2 \cdot \sin\vartheta \cdot \cos\vartheta =$$

$$= -\frac{\mu_0 \omega m}{4\pi R^2} \cdot \sin(2\vartheta)$$

$$E_\vartheta = -\frac{\partial\phi}{\partial\vartheta} \Rightarrow \phi = -\int E_\vartheta \, d\vartheta =$$

$$= \frac{\mu_0 \omega m}{4\pi R^2} \cdot \int \sin(2\vartheta)\, d\vartheta$$

$$= -\frac{\mu_0 \cdot \omega m}{4\pi R^2} \cdot \frac{1}{2}\cos(2\vartheta) + \text{konst.}$$

$$= \frac{\mu_0 \omega m}{4\pi R^2} \left(-\cos^2\vartheta + \frac{1}{2}\right) + \text{konst.}$$

Neobvezno:

Konstanto lahko dobeime tako, da bo krogla nevtralna, t.j. da bo $\langle\phi\rangle = 0$.

$\Downarrow$

$$\boxed{\phi = \frac{\mu_0 \omega m}{4\pi R^2}\left(\frac{1}{3} - \cos^2\vartheta\right)}$$

③ a) Iz enačb $\vec{j} = \sigma \vec{E} = -\sigma \vec{\nabla} \phi$

in $\vec{\nabla} \vec{j} = 0$ (v stac. stanju) sledi

$$\boxed{\Delta \phi = 0}$$

- Robni pogoj na površini je

$$\vec{n} \cdot \vec{j} = 0, \Rightarrow -\sigma \cdot \vec{n} \cdot \vec{\nabla} \phi = 0,$$

v krogelnih koordinatah

$$\boxed{\left.\frac{\partial \phi}{\partial r}\right|_{r=R} = 0}$$

- Ostala robna pogoja:

$$\boxed{\begin{array}{l} \phi(\vartheta = \alpha) = U \\ \phi(\vartheta = 0) = 0 \end{array}}$$

Problema se lotimo s separacijo spremenljivk:

$$\phi = \phi_R(r) \cdot \phi_\vartheta(\vartheta)$$

$\phi_R(r) = \text{konst.}$ izpolnjuje vse robne pogoje, zato poskusimo brez odvisnosti od r.

$$\phi = \phi_\vartheta(\vartheta)$$

Laplaceova enačba:

$$\Delta\phi = \frac{1}{r^2 \sin^2\vartheta}\,\partial_\vartheta\left(\sin\vartheta\,\frac{\partial\phi}{\partial\vartheta}\right) = 0$$

$$\Downarrow$$

$$\sin\vartheta\,\frac{\partial\phi}{\partial\vartheta} = \text{konst.}$$

$$\frac{\partial\phi}{\partial\vartheta} = \frac{C_1}{\sin\vartheta}$$

$$\phi(\vartheta) = \int \frac{C_1}{\sin\vartheta}\,d\vartheta = \ln\left(\tan\frac{\vartheta}{2}\right)\cdot C_1 + C_2$$

Robni pogoji: $\phi\left(\frac{\pi}{2}\right) = 0 \rightarrow \underline{C_2 = 0}$

$$\phi(\alpha) = U \rightarrow \ln\left(\tan\frac{\alpha}{2}\right)\cdot C_1 = U$$

$$C_1 = \frac{U}{\ln\left(\tan\frac{\alpha}{2}\right)} \quad (C_1 < 0!)$$

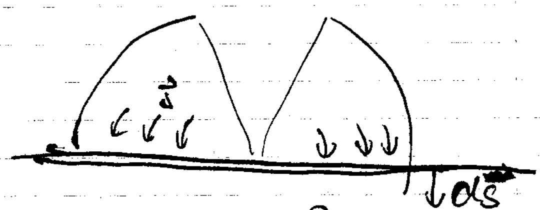
$$\boxed{\phi(\vartheta) = U\,\frac{\ln\left(\tan\frac{\vartheta}{2}\right)}{\ln\left(\tan\frac{\alpha}{2}\right)}}$$

<u>Opomba</u>: Gre za Legendrove funkcije druge vrste, ki jih zaradi singularnosti pri $\vartheta = 0, \pi$ običajno ne uporabljamo.

Rešitev s krogelnimi funkcijami bi bila formalno korektna, vendar zaradi komplicirane vrste precej neuporabna.

b) $$\vec{j} = -\sigma \cdot \vec{\nabla}\phi \underset{\phi = \phi(\vartheta)}{=} -\sigma \frac{1}{r}\frac{\partial \phi}{\partial \vartheta} \cdot \hat{e}_\vartheta$$

$$= \frac{-\sigma \cdot U}{r \cdot \sin\vartheta \cdot \ln\left(\tan\frac{\alpha}{2}\right)} \cdot \hat{e}_\vartheta$$



$$I = \int \vec{j}\, d\vec{S} = \int_0^R 2\pi r\, dr \cdot j(\vartheta = \tfrac{\pi}{2}) = \int_0^R 2\pi r\, dr \cdot \frac{-\sigma U}{\cancel{r} \underbrace{\sin\vartheta}_{=1} \cdot \ln\left(\tan\frac{\alpha}{2}\right)}$$

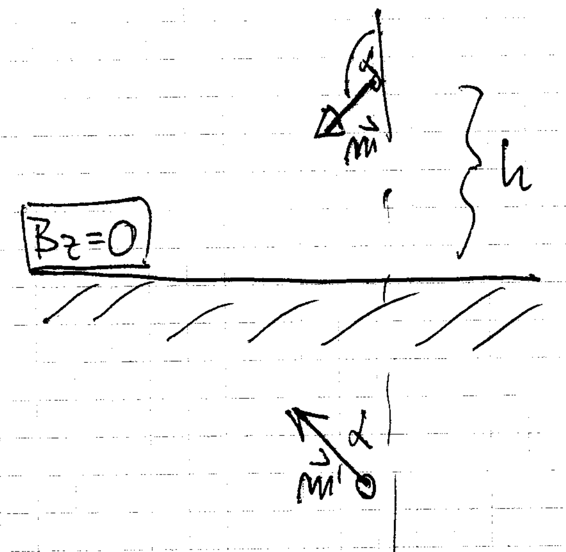
$$= \frac{-\sigma \cdot 2\pi R \cdot U}{\ln\left(\tan\frac{\alpha}{2}\right)}$$

$$\bar{R} = \frac{U}{I} = \frac{\left|\ln\left(\tan\frac{\alpha}{2}\right)\right|}{2\pi R \cdot \sigma}$$

4) a) Na površini diamagneta se ohranja $\underline{B_\perp}$ in $\underline{H_{\parallel}}$.

Ker je znotraj $\vec{B}=0$, bo zunaj torej $\boxed{\vec{n}\cdot\vec{B}=0}$

Robni pogoj za $\vec{B}$ je torej ravno ~~ta~~ drugačen, kot za $\vec{E}$.



Robni pogoj izpolnimo s simetričnim zrcaljenjem.

Brez izgube splošnosti dipol postavimo v x-z ravnino.

$$\vec{m} = m\cdot\begin{pmatrix}\sin\alpha\\0\\\cos\alpha\end{pmatrix} \qquad \vec{m}' = m\cdot\begin{pmatrix}\sin\alpha\\0\\-\cos\alpha\end{pmatrix}$$

$$\vec{r} = 2h\cdot\hat{e}_z$$

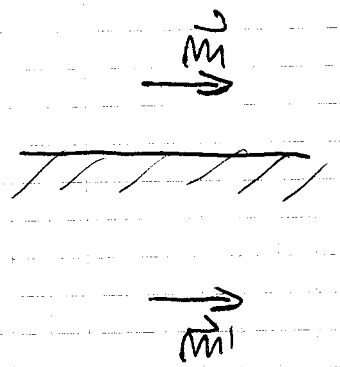
$$U = \frac{1}{2}\cdot\frac{\mu_0}{4\pi}\,\frac{-3(\vec{m}\vec{r})(\vec{m}'\vec{r}) + (\vec{m}\vec{m}')\cdot r^2}{|\vec{r}|^5}$$

Ker imamo polje samo v $\frac{1}{2}$ prostora.

$$U = \frac{\mu_0}{8\pi}\,\frac{-3m^2\cdot\cos\alpha(-\cos\alpha) + m^2(\sin^2\alpha-\cos^2\alpha)}{(2h)^3}$$

$$U = \frac{\mu_0}{8\pi} \frac{m^2}{8h^3} \left(2 \cdot \cos^2\alpha + \sin^2\alpha\right)$$

minimum pri $\boxed{\alpha = \frac{\pi}{2}}$



$$U = \frac{\mu_0 m^2}{64\pi h^3}$$

$$F = -\frac{\partial U}{\partial h} = \frac{3 \cdot \mu_0 m^2}{64\pi h^4} \overset{!}{=} Mg$$

$$\Downarrow$$

$$\boxed{h = \sqrt[4]{\frac{3m^2}{64\pi Mg}}}$$