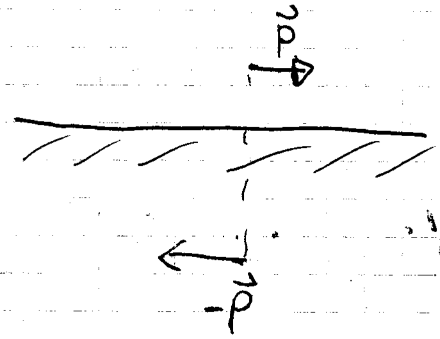
1) Valovanje se odbija z nasprotno fazo, tako da je $\vec{E}=0$ pri $z=0$.

$$\vec{E}_{sk.} = \vec{E}_0 \left(e^{i(-kz-\omega t)} - e^{i(kz-\omega t)}\right)\hat{e}_x =$$

$$= -2i\,\hat{e}_x \cdot \sin(kz)\, e^{-i\omega t} E_0$$

$$\vec{p} = \alpha \cdot \vec{E}_{sk.}(z=d) = -2i\,\hat{e}_x \sin(kd)\, e^{-i\omega t} \cdot \alpha E_0$$

Sipano valovanje se tudi odbija od plošče. Odbito valovanje opišemo z zrcaljenim virom, ki ima nasprotno fazo, zato da izpolni robni pogoj.



$$\vec{B} = \frac{\mu_0 \omega^2}{4\pi c \cdot r}(\vec{n}\times\vec{p})\left(e^{i(k(r-d\cdot\cos\vartheta)-\omega t)} - e^{i(k(r+d\cdot\cos\vartheta)-\omega t)}\right)$$

$$= \frac{\mu_0 \omega^2}{4\pi c \cdot r}(\vec{n}\times\vec{p})\cdot e^{i(kr-\omega t)}\,(-2i)\sin(kd\cdot\cos\vartheta)$$

$$\vec{m} \times \vec{p} = p\left(\vec{m} \times \hat{e}_x\right) = p \begin{pmatrix} \sin\vartheta \cos\varphi \\ \sin\vartheta \sin\varphi \\ \cos\vartheta \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$ 2)

$$= p\left(\cos\vartheta \cdot \hat{e}_y - \sin\vartheta \cos\varphi \cdot \hat{e}_z\right)$$

$$|\vec{m} \times \vec{p}|^2 = p^2 \cdot \left(\cos^2\vartheta + \sin^2\vartheta \cos^2\varphi\right)$$

$$\vec{S} = \frac{1}{2\mu_0} \vec{E} \times \vec{B}^* \underset{\substack{\uparrow \\ \text{Ravno valovanje}}}{=} \frac{c}{2\mu_0} |\vec{B}|^2 \cdot \vec{m} =$$

$$= \frac{c}{2\mu_0} \frac{\mu_0^2 \omega^4}{(4\pi)^2 c^2 r^2} \cdot 4\alpha^2 E_0^2 \cdot \sin^2(kd) \cdot \left(\cos^2\vartheta + \sin^2\vartheta \cos^2\varphi\right)$$

$$\cdot 4 \sin^2(kd\cos\vartheta) \cdot \vec{m}$$

$$\frac{dP}{d\Omega} = r^2 \cdot (\vec{S} \cdot \vec{m}) = \frac{\mu_0 \omega^4 \cdot \alpha^2 E_0^2}{2\pi^2 c} \cdot \sin^2(kd) \cdot \sin^2(kd\cos\vartheta)$$

$$\cdot \left(1 - \sin^2\vartheta \sin^2\varphi\right)$$

2) a) $\mathcal{L} = \frac{1}{2} m \dot{v}^2 + e \vec{v}\vec{A} =$

$$= \frac{1}{2} m(\dot{\rho}^2 + \rho^2 \dot{\varphi}^2 + \dot{z}^2) + e\rho\dot{\varphi}\,(A_0 \cdot \rho - A_1(\rho - R)^2 + A_1 \cdot z^2)$$

b) $p_\rho = \frac{\partial L}{\partial \dot{\rho}} = m\dot{\rho}$

$$\dot{p}_\rho\Big|_{\rho=R} = \frac{\partial L}{\partial \rho}\Big|_{\rho=R} = mR\dot{\varphi}^2 + 2A_0 \cdot eR\dot{\varphi}$$

φ)

$$m\ddot{\rho} = \dot{p}_\rho = 0 \Rightarrow mR\dot{\varphi}^2 + 2A_0 eR\dot{\varphi} = 0$$

$$\Rightarrow A_0 = -\frac{m\dot{\varphi}}{2e} = \boxed{-\frac{mv}{2eR}}$$

c) $p_z = \frac{\partial L}{\partial \dot{z}} = m\dot{z}$

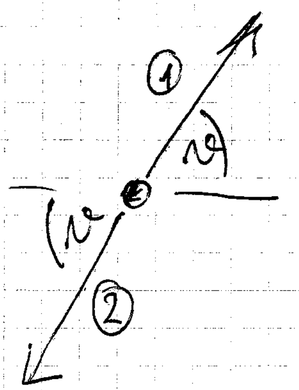
$$\dot{p}_z = \frac{\partial L}{\partial z} = \underbrace{e\rho\dot{\varphi}}_{v}\, 2A_1 z$$

$$m\ddot{z} = 2A_1 e v z$$

$$\omega_z = \sqrt{\frac{-2A_1 e v}{m}}$$

$$\Delta\varphi = \dot{\varphi} \cdot t_0 = \dot{\varphi} \cdot \frac{2\pi}{\omega_z} = 2\pi \frac{v/R}{\sqrt{-2A_1 e v/m}} = 2\pi\sqrt{\frac{mv}{-2A_1 e R^2}} = 2\pi\sqrt{\frac{A_0}{A_1 R}}$$

3) V težiščnem sistemu fotona odletita v nasprotnih smereh: Z (1) označimo tistega, ki ima $\underline{v_x>0}$



Za 2. velja:

$$v_x^{(2)} = -c\cdot\cos\vartheta$$

$$v_y^{(2)} = -c\cdot\sin\vartheta$$

Hitrosti sedaj transformirane v lab. sistem:

$$v_x' = \frac{v_x + v^0}{1+\frac{v_x v_0}{c^2}}$$

kjer je $v^0 = v(\text{piona}) = \beta\cdot c$

$$\gamma = \frac{E_\pi}{m_\pi c^2} = 2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{\frac{3}{4}}$$

Lahko bi transformirali tudi $\vec{p}$ ali $\vec{k}$.

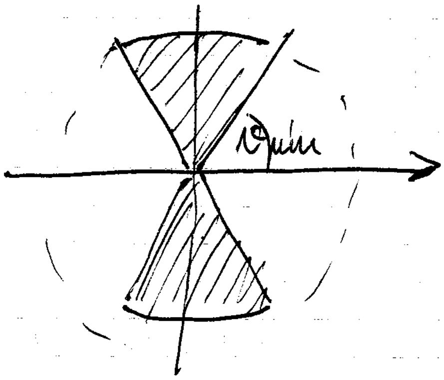
Za 2. foton velja

$$v_x^{(2)'} = \frac{v_x^{(2)}+\beta c}{(\ldots)} \Rightarrow \underline{v_x^{(2)'}>0} \text{ če je}$$

$$\underline{\underline{v_x^{(2)}+\beta c>0}}$$

Prvi foton ima zmeraj $\underline{v_x'>0}$.

$$-c\cdot\cos\beta+\beta c>0 \Rightarrow \boxed{\cos\vartheta<\beta} \qquad \underline{\vartheta_{min} = \arccos(\beta)}$$



Prvi foton mora torej imeti

$$\vartheta_{min} \leq \vartheta \leq \frac{\pi}{2}$$

Verjetnost: $$P = \frac{\int\limits_{\vartheta_{min} \leq \vartheta \leq \pi/2} d\Omega}{\int\limits_{0 \leq \vartheta \leq \pi/2} d\Omega} = \frac{\int_{\vartheta_{min}}^{\pi/2} 2\pi \sin\vartheta \, d\vartheta}{\int_0^{\pi/2} 2\pi \sin\vartheta \, d\vartheta}$$

$$= \frac{\int_0^{\cos\vartheta_{min}} d(\cos\vartheta)}{\int_0^1 d(\cos\vartheta)} = \cos(\vartheta_{min}) = \underline{\underline{\beta}}$$

$$\boxed{P = \frac{\sqrt{3}}{2}}$$

4) Robni pogoji za $\vec{E}$ in $\vec{B}$ na površini prevodnika so:

$$E_{\parallel}=0 \Rightarrow E_z=0$$

$$(\vec{n}\cdot\vec{\nabla}B_{\parallel})=0 \rightarrow (\vec{n}\cdot\vec{\nabla})B_z=0$$ ($\vec{n}$: normala na površino)

Slednji pogoj smo izpeljali na vajah.

<u>TM način</u>: iščemo E_z

$$\left(\Delta_t+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_z=0$$

Nastavek: $E_z=\bar{E}_z(r,\varphi)\cdot e^{i(kz-\omega t)}$

$$\Rightarrow \left(\Delta_t\underbrace{-k^2+\omega^2}_{p^2}\right)\bar{E}_z=0$$

$$\underline{\Delta_t\bar{E}_z+p^2\bar{E}_z=0}$$

Cilindrične koordinate:

$$\frac{1}{r}\partial_r(r\cdot\partial_r\bar{E}_z)+\frac{1}{r^2}\frac{\partial^2\bar{E}_z}{\partial\theta^2}+p^2\cdot\bar{E}_z=0$$

$$\bar{E}_z=\bar{\bar{E}}_z(r)\cdot e^{im\theta}$$

$$\Downarrow$$

$$r^2\partial_r^2\bar{\bar{E}}_z+r\cdot\partial_r\bar{\bar{E}}_z+(p^2r^2-m^2)\bar{\bar{E}}_z=0$$

rešitev: $\bar{\bar{E}}_z(r)=E^0\cdot J_m(pr)$

Robni pogoji:

$$J_m(pR)=0$$

$$\left.\begin{aligned} E_z(\theta=0)&=0 \\ E_z(\theta=\tfrac{\pi}{2})&=0 \end{aligned}\right\} \quad \bar{E}_z=\bar{E}_z(r)\cdot\sin(m\cdot\theta)$$

$$\underline{m=\cancel{0},2,4\ldots}$$

Trivialna rešitev

Najnižja rešitev: $J_2(pR)=0$

$$pR=x_{21}\cong\underline{\underline{5{,}14}}$$

$$\underline{\omega=\sqrt{k_z^2+\left(\frac{x_{21}}{R}\right)^2}\cdot c}$$

<u>TE</u> <u>način</u>: analogno, vendar je robni pogoj

$$\left.\begin{aligned} \left.\frac{\partial B_z}{\partial r}\right|_R&=0 \\ \left.\frac{\partial B_z}{\partial \varphi}\right|_{\varphi=0}&=0 \\ \left.\frac{\partial B_z}{\partial \varphi}\right|_{\varphi=\pi/2}&=0 \end{aligned}\right\}$$

$$B_z=B_z^0\cdot e^{im\varphi}\cdot J_m(pr)$$

$$J_m'(pR)=0 \Rightarrow p=\frac{x_{mn}'}{R}$$

$B_z(\varphi)$ izpeljemo z

$$B_z\sim\cos(m\cdot\varphi)$$

$$m=\underline{\underline{0}},2,4,\ldots$$

Prva ničla: $\left.\begin{aligned} x_{01}'&=3.8 \\ x_{21}'&=\underline{3.05}\end{aligned}\right\}$ $\underline{m=2}$ je najnižja rešitev. $\omega=\sqrt{k_z^2+\left(\frac{x_{21}'}{R}\right)^2}\,c$