Zbirka enačb, kot pripomoček pri kolokviju iz EMP

- Poissonova enačba

$$\nabla^2\varphi(\mathbf{r}) = -\frac{\rho(\mathbf{r})}{\varepsilon_0} \tag{1}$$

- Polje porazdelitven nabojev

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_{(V)} \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\, d^3\mathbf{r}' \tag{2}$$

$$\varphi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_{(V)} \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\, d^3\mathbf{r}' \tag{3}$$

- Polje električnega dipola

$$\varphi(\mathbf{r}) = \frac{\mathbf{p}\cdot\mathbf{r}}{4\pi\varepsilon_0 r^3} \tag{4}$$

$$\mathbf{E}(\mathbf{r}) = \frac{3(\mathbf{p}\cdot\mathbf{r})\mathbf{r} - \mathbf{p}r^2}{4\pi\varepsilon_0 r^5}. \tag{5}$$

$$\mathbf{p} = \int \mathbf{s}\rho(\mathbf{s})\, \mathrm{d}^3\mathbf{s} \tag{6}$$

- Biot - Savartov zakon

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{j}(\mathbf{r}')d^3\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} \tag{7}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_{(V)} \frac{\mathbf{j}(\mathbf{r}')\times(\mathbf{r}-\mathbf{r}')}{|\,\mathbf{r}-\mathbf{r}'\,|^3} d^3\mathbf{r}' \tag{8}$$

- Induktivnost

$$\Phi_i = \sum_k L_{ik} I_k \tag{9}$$

- Medsebojna induktivnost dveh zank

$$L_{ik} = \frac{\mu_0}{4\pi}\oint_{i,k} \frac{d\mathbf{l}_i d\mathbf{l}_k}{|\,\mathbf{r}(l_i)-\mathbf{r}(l_k)\,|} \tag{10}$$

- Polje magnetnega dipola

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{r}}{r^3} \tag{11}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{3\mathbf{r}\,(\mathbf{m}\cdot\mathbf{r}) - \mathbf{m}\,r^2}{r^5} \tag{12}$$

$$\mathbf{m} = \frac{1}{2}\int \mathbf{s}\times\mathbf{j}(\mathbf{s})\; d^3\mathbf{s} \tag{13}$$

- Električna prevodnost

$$\mathbf{j} = \sigma\mathbf{E} \tag{14}$$

- Maxwellove enačbe v vakuumu

$$\begin{aligned} \boldsymbol{\nabla}\cdot\mathbf{E} &= \frac{\rho}{\varepsilon_0} \qquad && \boldsymbol{\nabla}\cdot\mathbf{B} = 0 \\ \boldsymbol{\nabla}\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} &= 0 \qquad && \boldsymbol{\nabla}\times\mathbf{B} - \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t} - \mu_0\mathbf{j} = 0, \end{aligned} \tag{15}$$

- Maxwellove enačbe v snovi

$$\nabla \cdot \mathbf{D} = \rho \qquad \nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}. \tag{16}$$

- Susceptibilnost, permeabilnost

$$\mathbf{P} = \chi_E \mathbf{D} \qquad \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \tag{17}$$

$$\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} \qquad \mathbf{P} = \varepsilon_0 (\varepsilon - 1) \mathbf{E} \tag{18}$$

$$\mathbf{M} = \chi_M \mathbf{H} \qquad \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \tag{19}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu \mu_0} \qquad \mathbf{M} = (1 - \frac{1}{\mu}) \frac{\mathbf{B}}{\mu_0} \tag{20}$$

- Kontinuitetna enačba

$$\nabla \cdot \mathbf{j} = -\frac{\partial \rho}{\partial t} \tag{21}$$

- Poyntingov teorem

$$\frac{\partial}{\partial t} \int_{(V)} w \; d^3\mathbf{r} = - \oint_{(\partial V)} (\mathbf{P} \cdot \mathbf{n}) \; dS - \int_{(V)} (\mathbf{j} \cdot \mathbf{E}) \; d^3\mathbf{r} \tag{22}$$

$$\mathbf{P} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) \qquad w = \tfrac{1}{2} \varepsilon_0 E^2 + \tfrac{1}{2} \frac{B^2}{\mu_0} \tag{23}$$

- Ohranitev gibalne količine

$$\frac{\partial}{\partial t} \int_{(V)} g_i \; d^3\mathbf{r} = \oint_{(\partial V)} T_{ik} n_k \; dS - \int_{(V)} f_i \; d^3\mathbf{r} \tag{24}$$

$$\mathbf{g} = \varepsilon_0 (\mathbf{E} \times \mathbf{B}) \qquad T_{ik} = \varepsilon_0 E_i E_k - \tfrac{1}{2} \varepsilon_0 E^2 \delta_{ik} + \frac{1}{\mu_0} B_i B_k - \tfrac{1}{2} \frac{1}{\mu_0} B^2 \delta_{ik} \qquad \mathbf{f} = \rho \mathbf{E} + (\mathbf{j} \times \mathbf{B}) \tag{25}$$

- Potenciali

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t} \tag{26}$$

- Umeritvena transformacija

$$\begin{aligned} \mathbf{A}' &= \mathbf{A} + \nabla \chi \\ \varphi' &= \varphi - \frac{\partial \chi}{\partial t} \end{aligned} \tag{27}$$

- Lorentzova umeritev

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0 \tag{28}$$

- Sevalna umeritev

$$\nabla \cdot \mathbf{A} = 0 \tag{29}$$

- Riemann-Lorenzovi enačbi

$$\Box^2\varphi = \nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0} \tag{30}$$

$$\Box^2\mathbf{A} = \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{j} \tag{31}$$

- Retardiarani potenciali

$$\varphi(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c})\mathrm{d}^3\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} \qquad \mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{j}(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c})}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}^3\mathbf{r}' \tag{32}$$

- Lienard - Wiechertovi potenciali

$$\varphi(\mathbf{r},t) = \frac{e}{4\pi\varepsilon_0}\frac{1}{|\mathbf{r}-\mathbf{r}'(t')| - (\mathbf{r}-\mathbf{r}'(t'))\cdot\mathbf{v}(t')/c} \tag{33}$$

$$\mathbf{A}(\mathbf{r},t) = \frac{e}{4\pi\varepsilon_0 c^2}\frac{\mathbf{v}(t')}{|\mathbf{r}-\mathbf{r}'(t')| - (\mathbf{r}-\mathbf{r}'(t'))\cdot\mathbf{v}(t')/c} \tag{34}$$

- Sevalno polje transverzalnih tokov

$$\mathbf{A}(\mathbf{r},t) \simeq -\frac{\mathbf{r}}{r}\times\left(\frac{\mathbf{r}}{r}\times\frac{\mu_0}{4\pi r}\left(\int_V \mathbf{j}(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c})\ d^3\mathbf{r}'\right)\right) \tag{35}$$

$$\mathbf{E}(\mathbf{r},t) = -\frac{\partial\mathbf{A}(\mathbf{r},t)}{\partial t} \simeq \frac{\mathbf{r}}{r}\times\left(\frac{\mathbf{r}}{r}\times\frac{\mu_0}{4\pi r}\int_V \frac{\partial\mathbf{j}(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c})}{\partial t}\ d^3\mathbf{r}'\right) \tag{36}$$

$$\mathbf{B}(\mathbf{r},t) = \boldsymbol{\nabla}\times\mathbf{A}(\mathbf{r},t) \simeq -\frac{\mathbf{r}}{r}\times\frac{\mu_0}{4\pi c r}\int_V \frac{\partial\mathbf{j}(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c})}{\partial t}d^3\mathbf{r}' \tag{37}$$

- Hertzov vektor

$$\mathbf{A} = \mu_0\frac{\partial\mathbf{Z}}{\partial t} \qquad \varphi = -\frac{1}{\varepsilon_0}\boldsymbol{\nabla}\cdot\mathbf{Z} \tag{38}$$

$$\mathbf{j} = \frac{\partial\mathbf{J}}{\partial t} \qquad \rho = -\boldsymbol{\nabla}\cdot\mathbf{J} \tag{39}$$

$$\nabla^2\mathbf{Z} - \frac{1}{c^2}\frac{\partial^2\mathbf{Z}}{\partial t^2} = -\mathbf{J} \tag{40}$$

$$\mathbf{Z}(\mathbf{r},t) = \frac{1}{4\pi}\int_V \frac{\mathbf{J}(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c})\ d^3\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} \tag{41}$$

- Dipolno sevanje

$$\mathbf{B} \cong -\frac{\mu_0}{4\pi c|\mathbf{r}|}\left(\frac{\mathbf{r}}{|\mathbf{r}|}\times\ddot{\mathbf{p}}(t-\frac{|\mathbf{r}|}{c})\right) \qquad \mathbf{E} \cong \frac{\mu_0}{4\pi|\mathbf{r}|}\left(\frac{\mathbf{r}}{r}\times\left(\frac{\mathbf{r}}{r}\times\ddot{\mathbf{p}}(t-\frac{r}{c})\right)\right) \tag{42}$$

- Ravno EM valovanje v vakuumu

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \qquad \mathbf{B}(\mathbf{r},t) = \mathbf{B}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \tag{43}$$

$$\mathbf{n}\times\mathbf{E}_0 = c\mathbf{B}_0 \qquad \mathbf{n} = \frac{\mathbf{k}}{k} \qquad \omega = kc \tag{44}$$

- Euler - Lagrangeova enačba

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{\mathbf{r}}}\right) - \frac{\partial L}{\partial \mathbf{r}} = 0 \tag{45}$$

- Lagrangeova funkcija nabitega delca v polju

$$L(\mathbf{r}(t), \dot{\mathbf{r}}(t), t) = \frac{1}{2}m\dot{\mathbf{r}}(t)^2 - e\varphi(\mathbf{r}(t), t) + e\dot{\mathbf{r}}(t) \cdot \mathbf{A}(\mathbf{r}(t), t) \tag{46}$$

- Hamiltonova funkcija

$$\mathcal{H}(\mathbf{p},\mathbf{r},t) = \dot{\mathbf{r}} \cdot \mathbf{p} - L = \frac{1}{2m}(\mathbf{p}(t) - e\mathbf{A}(\mathbf{r}, t))^2 + e\varphi(\mathbf{r}, t) \tag{47}$$

- Hamiltonove enačbe gibanja

$$\dot{\mathbf{r}}(t) = \frac{\partial H}{\partial \mathbf{p}} \qquad \dot{\mathbf{p}}(t) = -\frac{\partial H}{\partial \mathbf{r}} \tag{48}$$

- Lorentzova transformacija

$$\begin{aligned}
x' &= \gamma(x - vt) && (49)\\
y' &= y && (50)\\
z' &= z && (51)\\
t' &= \gamma(t - \frac{v}{c^2}x) && (52)\\
\gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} && (53)
\end{aligned}$$

$$\begin{aligned}
A'_x &= \gamma\left(A_x - \beta\frac{\varphi}{c}\right)\\
A'_y &= A_y\\
A'_z &= A_z\\
\varphi' &= \gamma(\varphi - \beta c A_x) && (54)
\end{aligned}$$

$$\begin{aligned}
E'_x &= E_x\\
E'_y &= \gamma(E_y - vB_z)\\
E'_z &= \gamma(E_z + vB_y) && (55)
\end{aligned}$$

$$\begin{aligned}
B'_x &= B_x\\
B'_y &= \gamma(B_y + \frac{v}{c^2}E_z)\\
B'_z &= \gamma(B_z - \frac{v}{c^2}E_y) && (56)
\end{aligned}$$