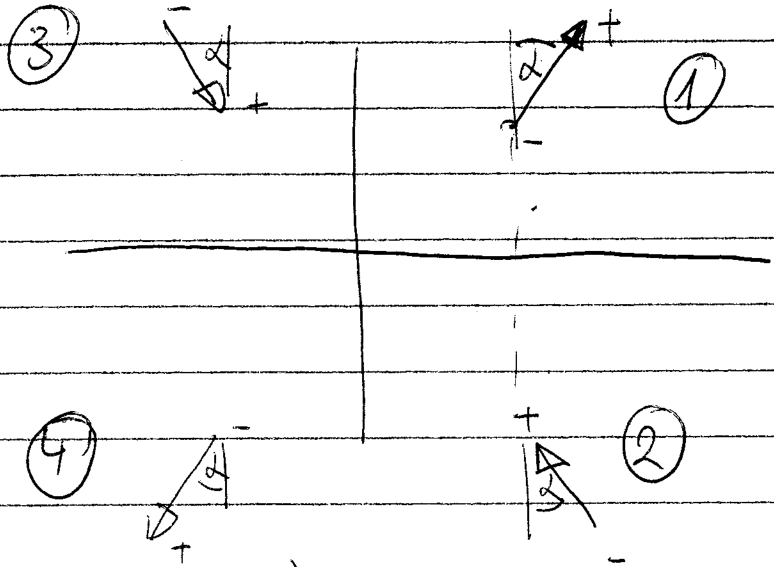
1.) a)



$$\vec{p}_1 = p \cdot \begin{pmatrix} \sin\alpha \\ 0 \\ \cos\alpha \end{pmatrix} \qquad \vec{p}_2 = p \cdot \begin{pmatrix} -\sin\alpha \\ 0 \\ \cos\alpha \end{pmatrix}$$

$$\vec{p}_3 = p \begin{pmatrix} \sin\alpha \\ 0 \\ -\cos\alpha \end{pmatrix} \qquad \vec{p}_4 = p \begin{pmatrix} -\sin\alpha \\ 0 \\ -\cos\alpha \end{pmatrix}$$

b)

$$W = \frac{1}{2}\left(W_{12} + W_{13} + W_{14}\right)$$

$$\hat{r}_{12} = \hat{e}_z$$

$$W_{12} = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}_1\vec{p}_2 - 3(\hat{r}_{12}\vec{p}_1)(\hat{r}_{12}\vec{p}_2)}{r_{12}^3} =$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{p^2 \cdot (\cos^2\alpha - \sin^2\alpha) - 3\cos^2\alpha}{(2a)^3} =$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{p^2}{(2a)^3} \left(-\sin^2\alpha - 2\cos^2\alpha\right)$$

Analogno: $W_{13} = \frac{1}{4\pi\varepsilon_0} \frac{p^2}{(2a)^3} \left(-\cos^2\alpha - 2\sin^2\alpha\right)$

$$\hat{r}_{34} \cdot \vec{p}_1 = p \cdot \cos\left(\alpha - \frac{\pi}{4}\right)$$

$$\hat{r}_{34} \cdot \vec{p}_2 = -p \cdot \cos\left(\alpha - \frac{\pi}{4}\right)$$

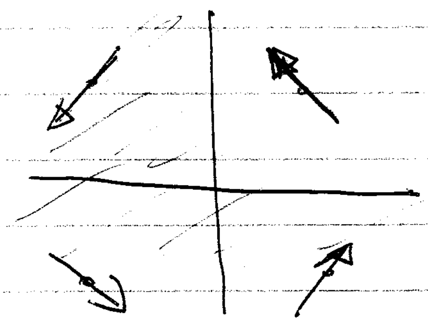
$$W_{14} = \frac{1}{4\pi\varepsilon_0 \left(\sqrt{2}\cdot 2a\right)^3}\left(-p^2 + 3p^2 \cdot \cos^2\left(\alpha - \frac{\pi}{4}\right)\right)$$

$$W = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{p^2}{(2a)^3}\cdot\left(-3 - \frac{1}{(\sqrt{2})^3} + \frac{3}{(\sqrt{2})^3}\cos^2\left(\alpha - \frac{\pi}{4}\right)\right)$$

c) Minimalna bodo je $\cos\left(\alpha - \frac{\pi}{4}\right) = 0$

$$\boxed{\alpha = -\frac{\pi}{4}} \quad \text{ali} \quad \boxed{\alpha = \frac{3\pi}{4}}$$

Rešitev.



2) a) $\vec{A} = \frac{\mu_0 e R^2}{20\pi} \frac{\vec{\omega} \times \vec{r}}{r^3}$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \frac{\mu_0 e R^2}{20\pi} \underbrace{\vec{\nabla} \times \left(\vec{\omega} \times \frac{\vec{r}}{r^3}\right)}$$

$\vec{\omega} = \omega \cdot \hat{e}_z$ $$-\underbrace{(\vec{\omega}\cdot\vec{\nabla})\frac{\vec{r}}{r^3}} + \vec{\omega}\underbrace{\left(\vec{\nabla}\cdot\frac{\vec{r}}{r^3}\right)}_{=0}$$

$$-\omega \frac{\partial}{\partial z}\frac{\vec{r}}{r^3} = -\omega\frac{\hat{e}_z}{r^3} + 3\omega\frac{\vec{r}\cdot z}{r^5}$$

$$\vec{B} = \boxed{\frac{\mu_0 e R^2}{20\pi} \frac{3\hat{e}_r(\vec{\omega}\cdot\hat{e}_r) - \vec{\omega}}{r^3}}$$

b) $$W_M = \int dV \frac{1}{2\mu_0} \cdot B^2 = \frac{1}{2\mu_0}\int_R^\infty r^2\, dr \cdot 2\pi \cdot \int_{-1}^{1} d(\cos\vartheta) \cdot$$

$$\cdot \left(\frac{\mu_0 e R^2}{20\pi}\right)^2 \frac{9\omega^2\cos^2\vartheta - 6\omega^2\cos^2\vartheta + \omega^2}{r^6}$$

$\vec{\omega}\cdot\hat{e}_r = \omega \cdot \cos\vartheta$

$$W_M = \frac{1}{2\mu_0} \cdot \frac{\mu_0^2 e^2 R^4}{400\pi^2} \cdot 2\pi\, \omega^2 \underbrace{\int_R^\infty \frac{1}{r^4} dr}_{\frac{1}{3R^3}} \cdot \underbrace{\int_{-1}^{1} (3\cos^2\vartheta + 1)\, d(\cos\vartheta)}_{4}$$

$$= \frac{\mu_0 e^2 R \omega^2}{300\pi}$$

c) $\vec{E} = -\underbrace{\vec{\nabla}\varphi}_{\text{konstantno}} - \frac{\partial \vec{A}}{\partial t}$

$$\vec{E}^{(1)} = -\frac{\partial \vec{A}}{\partial t} = -\frac{\mu_0 e R^2}{20\pi}\left(\dot{\vec{\omega}} \times \frac{\vec{r}}{r^3}\right) = \frac{\mu_0 e R^2}{20\pi}\gamma\left(\vec{\omega} \times \frac{\vec{r}}{r^3}\right)$$

d) $\vec{P} = \frac{1}{\mu_0} \cdot \vec{E} \times \vec{B}$

radiala komponenta: $P_r = \frac{1}{\mu_0} \cdot (\underbrace{E_\vartheta B_\varphi}_{=0} - E_\varphi B_\vartheta)$

$$E_\varphi = \frac{\mu_0 e R^2}{20\pi} \gamma \cdot \omega \cdot \frac{1}{r^2} \cdot \sin\vartheta$$

$$B_\vartheta = \frac{\mu_0 e R^2}{20\pi} \omega \frac{\sin\vartheta}{r^3}$$

$$P_r\Big|_{r=R} = -\frac{\mu_0 e^2 R^4}{400\pi^2} \cdot \gamma \cdot \omega^2 \cdot \sin^2\vartheta \frac{1}{R^5}$$

$$= -\frac{\mu_0 e^2}{400\pi^2 R} \omega^2 \cdot \gamma \cdot \sin^2\vartheta$$

$$P_{tot.} = 2\pi R^2 \cdot \int_{-1}^{1} d(\cos\vartheta)\, P_r(\vartheta) \cdot$$

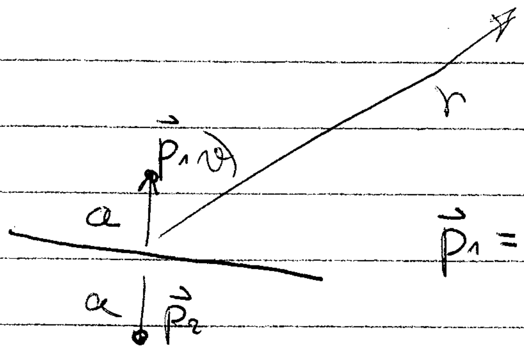
$$= -\frac{\mu_0 e^2}{200\pi \cancel{R}} \cdot R^{\cancel{2}} \cdot \omega^2 \cdot \gamma \cdot \underbrace{\int_{-1}^{1} (1-\cos^2\vartheta)\, d\cos\vartheta}_{2-\frac{2}{3}=\frac{4}{3}}$$

$$= -\frac{\mu_0 e^2}{150\pi} R\,\omega^2 \gamma$$

$$W = \int_0^\infty P_{tot.}\, dt = \int_0^\infty \left(-\frac{\mu_0 e^2 R}{150\pi} \cdot \gamma \cdot \omega_0^2 \cdot e^{-2\gamma t}\right) dt$$

$$= -\frac{\mu_0 e^2 R\, \omega_0^2}{300\pi} \quad ✓$$

3) a)



$$\vec{p}_1 = \vec{p}_2 = \alpha \cdot \hat{e}_z \cdot E_0 \cdot e^{-i\omega t}$$

$$\vec{B} = -\frac{\mu_0}{4\pi c \cdot r} \left(\hat{e}_r \times \hat{e}_z\right) \cdot \left(-\omega^2\right) \alpha \cdot E_0 \cdot \left(e^{i(kr - ka\cos\vartheta - \omega t)} + e^{i(kr + ka\cos\vartheta - \omega t)}\right)$$

$$= + \frac{\mu_0}{4\pi c \cancel{r}} \sin\vartheta \cdot \hat{e}_\varphi \left(-\omega^2\right) \alpha E_0 \, 2 \cdot \cos\left(ka\cos\vartheta\right) \cdot e^{i(kr - \omega t)}$$

$$\frac{\partial P}{\partial \Omega} = r^2 \cdot \bar{P}_r = r^2 \cdot \frac{1}{2\mu_0} \cdot c|B|^2 =$$

$$a = \frac{\lambda}{2} = \frac{\pi}{k} \quad \Downarrow$$

$$= \frac{\mu_0^{\cancel{2}} \cdot \cancel{c}}{4\pi^2 \cdot c^{\cancel{2}} \cdot 2\cancel{\mu_0}} \cdot \alpha^2 E_0^2 \omega^4 \sin^2\vartheta \cdot \cos^2\left(\pi\cos\vartheta\right)$$

b) $\frac{\partial P}{\partial \Omega} = 0$ pri $\vartheta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$