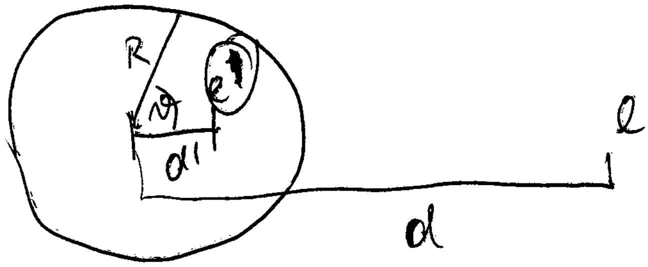
1) Najprej rešimo problem za ozemljeno kroglo (znan z Noj), potem pa bomo dodali naboj, ki jo naredi nevtralno.



R.P.: $\underline{\phi(R,\vartheta)=0}$

$$\phi(R,\vartheta) = \frac{1}{4\pi\varepsilon_0}\left(\frac{e}{\sqrt{d^2+R^2-2dR\cdot\cos\vartheta}} + \frac{e'}{\sqrt{d'^2+R^2-2d'R\cos\vartheta}}\right)$$

$$=0$$

$$\frac{d^2+R^2-2dR\cdot\cos\vartheta}{e^2} = \frac{d'^2+R^2-2d'R\cos\vartheta}{(e')^2} \quad \forall\vartheta$$

$$\frac{d^2+R^2}{2dR} = \frac{d'^2+R^2}{2d'R} \qquad \frac{e^2}{2dR} = \frac{e'^2}{2d'R}$$

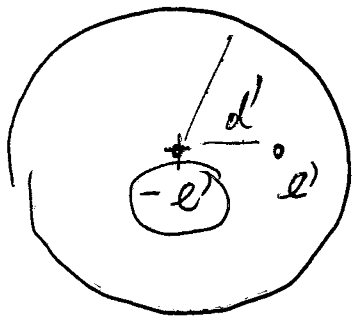
$$\frac{d}{R}+\frac{R}{d} = \frac{d'}{R}+\frac{R}{d'}$$

$\downarrow$

Netrivialna reš. $\frac{d}{R} = \frac{R}{d'}$ $\qquad \boxed{d' = \frac{R^2}{d}}$ $\qquad e' = -e\sqrt{\frac{d'}{d}} = \underline{\underline{-e\frac{R}{d}}}$

Nevtralnost dosežemo z dodatnim zrc. nabojem,

ki pusti $\phi(R,\vartheta)=$ konst. očitno ne prekrši:



$-e'$ v središču krogle.

$$F = \frac{1}{4\pi\varepsilon_0}\left(\frac{e'e}{|d-d'|^2} + \frac{(-e')e}{d^2}\right)$$

$$= \frac{e^2}{4\pi\varepsilon_0}\,\frac{R}{d}\left(\frac{-1}{\left(d-\frac{R^2}{d}\right)^2} + \frac{1}{d^2}\right) =$$

$$= \frac{e^2}{4\pi\varepsilon_0}\,Rd\left(\frac{-1}{(d^2+R^2)^2} + \frac{1}{d^4}\right) < 0$$, torej privlačna.

2) $\vec{E} = \frac{-e}{4\pi\varepsilon_0 r^2} \hat{e}_r$

$\vec{B} = B \cdot \hat{e}_z$

$$\vec{g} = \frac{1}{c^2}\vec{P} = \varepsilon_0 \vec{E} \times \vec{B} = \varepsilon_0 \frac{-e}{4\pi\varepsilon_0 r^2} \cdot B \cdot \underbrace{\hat{e}_r \times \hat{e}_z}_{-\sin\vartheta \cdot \hat{e}_\varphi}$$

$$\vec{\gamma} = \vec{r} \times \vec{g} = \frac{eB}{4\pi r^2} \cdot r \cdot \sin\vartheta \cdot \underbrace{\hat{e}_r \times \hat{e}_\varphi}_{-\hat{e}_\vartheta}$$

Zaradi simetrije bo

$$\vec{\Gamma} = \Gamma_z \cdot \hat{e}_z$$

$$\Gamma_z = \int_{R_1}^{R_2} dr \cdot 2\pi r^2 \int_0^\pi \sin\vartheta \, d\vartheta \, \frac{eB}{4\pi r} \sin\vartheta \cdot \sin\vartheta$$

$$= \frac{eB}{2}\left(\frac{R_2^2}{2} - \frac{R_1^2}{2}\right) \underbrace{\int_0^\pi \sin^3\vartheta \, d\vartheta}_{\int_{-1}^{1}(1-\cos^2\vartheta)\, d(\cos\vartheta) = \frac{4}{3}} = \frac{eB}{3}(R_2^2 - R_1^2)$$

$$\omega = \frac{\Gamma}{J} = \frac{eB(R_2^2 - R_1^2)}{3J}$$

3) a) $\vec{E} = \hat{e}_y \cdot \sin(\varkappa z)\, e^{i(kx-\omega t)} =$

$$= \hat{e}_y \cdot \frac{1}{2i}\left(e^{i\varkappa z} - e^{-i\varkappa z}\right) e^{i(kx-\omega t)}$$

gre torej za superpozicijo valovnih vektorjev

$$\begin{pmatrix} k \\ 0 \\ \varkappa \end{pmatrix} \text{ in } \begin{pmatrix} k \\ 0 \\ -\varkappa \end{pmatrix}$$

Veljati mora $\vec{k}^2 = \frac{\omega^2}{c^2}$, torej $\underline{\omega^2 = c^2(k^2+\varkappa^2)}$

Robni pogoj $\vec{E}_{\parallel} = 0$ pa je izpolnjen z

$$\varkappa \cdot d = n \cdot \pi$$

$$\underline{\underline{\varkappa = n\frac{\pi}{d}}}$$

Osnovna rešitev je $\varkappa = \frac{\pi}{d}$, $\omega = c\sqrt{k^2 + \left(\frac{\pi}{d}\right)^2}$

b) V mediju velja:

$$\vec{k}^2 = \varepsilon_0 \mu_0 \varepsilon(\omega) \cdot \omega^2$$

$$k^2 + \varkappa^2 = \varepsilon_0 \mu_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right) \omega^2$$

$$k^2 + \varkappa^2 = \overbrace{\varepsilon_0 \mu_0}^{\frac{1}{c^2}} \left(\omega^2 - \omega_p^2\right)$$

$$\omega^2 = \omega_p^2 + c^2(k^2 + \varkappa^2)$$

$$\boxed{\omega_{min.} = \sqrt{\omega_p^2 + c^2 \pi^2 / d^2}}$$