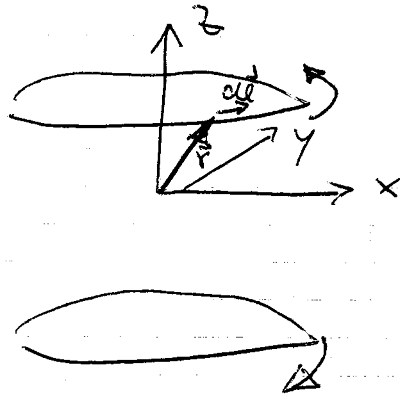
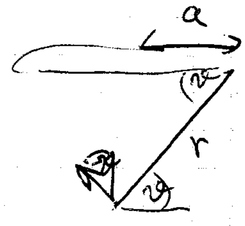
1)





$$\cos\vartheta = \frac{a}{r}$$

$$\vec{B}^{(1)} = \frac{\mu_0}{4\pi} \cdot \oint \frac{I\, d\vec{l} \times \vec{r}}{|\vec{r}|^3}$$

$$B_z^{(1)} = \frac{\mu_0}{4\pi} \cdot \int_0^{2\pi} \frac{I \cdot a \cdot d\varphi \cdot r \cdot \cos\vartheta}{r^3} = \frac{\mu_0}{2} \frac{I a^2}{r^3}$$

$$= \frac{\mu_0 I a^2}{2\left(\sqrt{a^2 + \left(\frac{d}{2} - z\right)^2}\right)^3}$$

Analogue: $$B_z^{(2)} = -\frac{\mu_0 I a^2}{2\left(\sqrt{a^2 + \left(\frac{d}{2} + z\right)^2}\right)^3}$$

$$\frac{\partial B_z^{(1)}}{\partial z} = \frac{\mu_0 I a^2}{2} \frac{3 \cdot \left(\frac{d}{2} - z\right)}{\left(a^2 + \left(\frac{d}{2} - z\right)^2\right)^{5/2}}$$

$$\frac{\partial B_z^{(2)}}{\partial z} = -\frac{\mu_0 I a^2}{2} \frac{-3\left(\frac{d}{2} + z\right)}{\left(a^2 + \left(\frac{d}{2} + z\right)^2\right)^{5/2}}$$

$$\left.\frac{\partial B_z}{\partial z}\right|_{z=0} = \left.\frac{\partial B_z^{(1)}}{\partial z}\right|_{z=0} + \left.\frac{\partial B_z^{(2)}}{\partial z}\right|_{z=0} = \frac{3\mu_0 I a^2}{2} \frac{d}{\left(a^2 + \left(\frac{d}{2}\right)^2\right)^{5/2}}$$

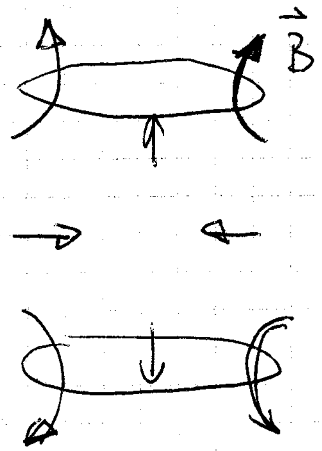
Maximum: $$\frac{\partial}{\partial d}\left(\left.\frac{\partial B_z}{\partial z}\right|_0\right) = 0$$

$$0 = \frac{3\mu_0 I a^2}{2}\left(\frac{1}{\left(a^2 + \left(\frac{d}{2}\right)^2\right)^{5/2}} - \frac{5d\,\frac{d}{4}}{\left(a^2 + \left(\frac{d}{2}\right)^2\right)^{7/2}}\right) = \ldots \frac{a^2 + \frac{d^2}{4} - \frac{5d^2}{4}}{(\ldots)}$$

Rešitev: $\underline{\underline{a=d}}$

$$\left.\frac{\partial B_z}{\partial z}\right|_0 = \frac{3\mu_0 I a^2}{2}\,\frac{a}{\left(a^2+\left(\frac{a}{2}\right)^2\right)^{5/2}} = \frac{3}{2}\left(\frac{4}{5}\right)^{5/2}\frac{\mu_0 I}{a^2} = A$$

b)



Iz simetrije vidimo, da je:

B_z $\underline{\text{lih}}$ v z in $\underline{\text{sod}}$ v x,y

$B_{x,y}$: $\underline{\text{sod}}$ v z in $\underline{\text{lih}}$ v x,y

Hkrati mora biti zaradi simetrije $\underline{B_\varphi = 0}$.

Torej pridejo v pošev samo členi:

$$\vec{B}(x,y,z) = A\cdot z\cdot \hat{e}_z + C\cdot x\,\hat{e}_x + C\cdot y\,\hat{e}_y + O($$

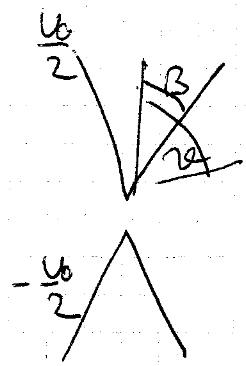
$$\operatorname{div}\vec{B}=0 \Rightarrow \frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0$$

$$C+C+A=0$$

$$\underline{C=-A/2}$$

$$\underline{\underline{\vec{B}= A\cdot z\cdot\hat{e}_z-\frac{1}{2}A\cdot\rho\cdot\hat{e}_\rho+O(r^3)}},\ A=\frac{3}{2}\left(\frac{4}{5}\right)^{5/2}\frac{\mu_0 I}{a^2}$$

2)



Robni pogoji so odvisni samo od ϑ, zato uporabimo nastavek:

$$\phi = \phi(\vartheta)$$

$$\Delta\phi = \frac{1}{r^2 \sin\vartheta} \frac{\partial}{\partial\vartheta}\left(\sin\vartheta \frac{\partial\phi}{\partial\vartheta}\right) = 0$$

$$\sin\vartheta \frac{\partial\phi}{\partial\vartheta} = C$$

$$\phi = C \cdot \int \frac{d\vartheta}{\sin\vartheta} + D = C \cdot \ln\left(\tan\frac{\vartheta}{2}\right) + D$$

Zaradi simetrije: $\phi\left(\vartheta = \frac{\pi}{2}\right) = 0 \Rightarrow \underline{D = 0}$

Robni pogoj: $\phi(\beta) = C \cdot \ln\left(\tan\frac{\beta}{2}\right) = \frac{U_0}{2}$

$\mathcal{O}(r^3)$

$$C = \frac{U_0}{2 \cdot \ln\left(\tan\frac{\beta}{2}\right)}$$

$$\rightarrow \underline{\phi(\vartheta) = \frac{U_0}{2} \frac{\ln\left(\tan\left(\frac{\vartheta}{2}\right)\right)}{\ln\left(\tan\left(\frac{\beta}{2}\right)\right)}}$$

$$\vec{E} = -\vec{\nabla}\phi = -\frac{1}{r}\frac{\partial\phi}{\partial\vartheta} \cdot \hat{e}_\vartheta = \underline{-\frac{1}{r} \cdot \frac{C}{\sin\vartheta} \hat{e}_\vartheta}$$

$\frac{\pi}{2}$

3) $\phi|_R = U \cdot \cos^2\vartheta + U_0$

$\underline{r>R}$: $\phi^+ = \frac{B_0}{r} \cdot 1 + \frac{B_1}{r^2} \cdot \cos\vartheta + \frac{B_2}{r^3} \cdot \frac{3\cos^2\vartheta - 1}{2} + \ldots$

$\phi^+(r=R) = U \cdot \cos^2\vartheta + U_0$

$\underline{B_0 = 0}$, da je krogla električno <u>nevtralna</u>.

$\Downarrow$

$\underline{B_1 = 0}$ $\quad B_2 = \frac{2}{3} U \cdot R^3 \quad B_{3,4,\ldots} = 0$

$$\phi\left(r, \vartheta = \frac{\pi}{2}\right) = \frac{2}{3} U \frac{R^3}{r^3}\left(-\frac{1}{2}\right)$$

$$E_r = -\frac{\partial\phi}{\partial r} = U \cdot \frac{R^3}{r^4}$$

$\underline{r<R}$: $\phi^- = A_0 + A_1 r \cdot \cos\vartheta + A_2 r^2 \frac{3\cos^2\vartheta - 1}{2}$

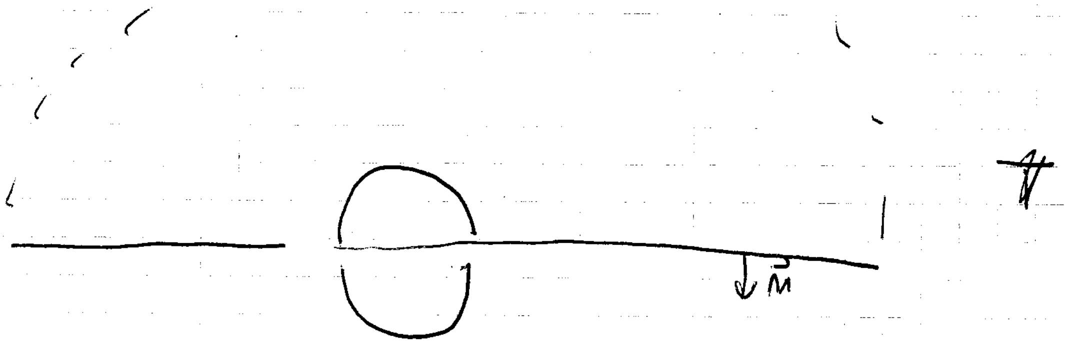
$\phi^-(r=R) = U \cdot \cos^2\vartheta + U_0$

$\Rightarrow A_2 = \frac{2}{3R^2} \cdot U$

$$\phi\left(r, \vartheta = \frac{\pi}{2}\right) = \frac{2}{3} U \frac{r^2}{R^2}\left(-\frac{1}{2}\right)$$

$$E_r = -\frac{\partial\phi}{\partial r} = \frac{2}{3} U \frac{r}{R^2}$$

E_z oziroma E_ϑ je pri $\vartheta = \frac{\pi}{2}$ enak 0, zaradi simetrije.



Sila na zgornjo polovico:

$$\vec F = \oint \overleftrightarrow{T}\cdot \vec n \, dS \quad , \quad F_z = \oint -T_{zz}\, dS$$

$$T_{zz} = \varepsilon_0 \cdot \Big(\underbrace{E_z \cdot E_z}_{=0 \text{ pri } z=0} - \frac{1}{2} E^2\Big) = -\frac{1}{2}\varepsilon_0 E^2$$

$$F_z = \int \frac{1}{2}\varepsilon_0 E^2 \, dS = \frac{\varepsilon_0}{2}\int_R^\infty \left(U \frac{R^3}{r^4}\right)^2 2\pi r \, dr + \frac{\varepsilon_0}{2}\int_0^R \left(\frac{2U}{3}\frac{r}{R^2}\right)^2 \cdot 2\pi r \, dr$$

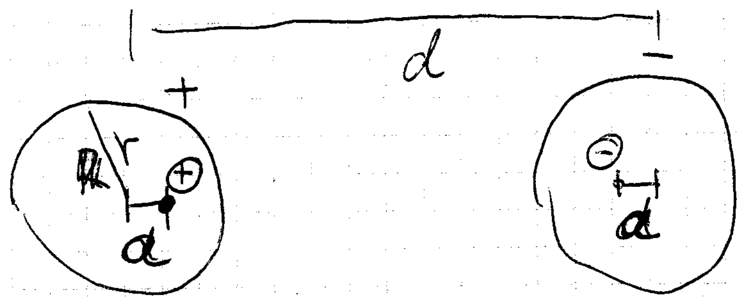
$$= \frac{\varepsilon_0 U^2 2\pi}{2}\left(\frac{R^6}{6 \cdot R^6} + \frac{4}{9}\frac{R^4}{4 \cdot R^4}\right) = \underline{\frac{5}{18}\varepsilon_0 \pi U^2}$$

Pozitiven predznak pomeni silo navzgor, torej je <u>odbojna</u>.

4) Velja $\vec{j} = \sigma \cdot \vec{E}$, $\vec{E} = -\vec{\nabla}\phi$

$$\underline{\Delta\phi = 0}$$

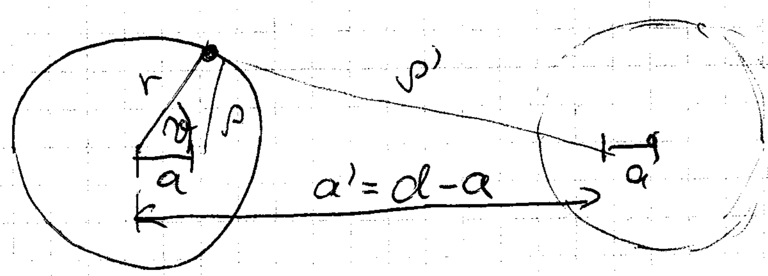
Ker $\vec{j}$ teče samo v ravnini x, y, je problem 2-dimenzionalen.



Naboj elektrode nadomestimo ~~z~~ s premico na razdalji a ~~d~~ od središča.

Potencial nabite premice je:

$$E_\rho(\rho) = \frac{(e/D)}{2\pi\varepsilon_0\rho} \qquad \phi(\rho) = -\frac{(e/D)}{2\pi\varepsilon_0} \cdot \ln\rho$$



$$\rho^2 = a^2 + r^2 - 2ra \cdot \cos\vartheta$$

$$\rho'^2 = a'^2 + r^2 - 2ra' \cos\vartheta$$

Potencial na levi elektrodi:

$$\phi_L(\vartheta) = -\frac{(e/D)}{2\pi\varepsilon_0}\cdot\left(\ln\rho - \ln\rho'\right) =$$

$$= -\frac{e/D}{2\pi\varepsilon_0}\cdot\ln\frac{\rho}{\rho'} =$$

$$= -\frac{e/D}{2\pi\varepsilon_0}\cdot\frac{1}{2}\ln\frac{a^2+r^2-2ra\cdot\cos\vartheta}{a'^2+r^2-2ra'\cos\vartheta}$$

mora biti neodvisen od ϑ, kar je izpolnjeno, če je

$$\frac{a^2+r^2}{\cancel{2}ra} = \frac{a'^2+r^2}{\cancel{2}ra'}$$

$$\frac{a}{r}+\frac{r}{a} = \frac{a'}{r}+\frac{r}{a'}$$

$$\downarrow$$

$$\frac{a'}{r} = \frac{r}{a} \rightarrow \boxed{a' = \frac{r^2}{a}}$$

$$a' = d-a \rightarrow d-a = \frac{r^2}{a}$$

$$a^2 - da + r^2 = 0$$

$$a = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - r^2}$$

$$a' = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 - r^2}$$

Izračunajmo sedaj:

$$\phi_L(\vartheta=0) = -\frac{e/D}{2\pi\varepsilon_0}\ln\frac{r-a}{a'-r} =$$

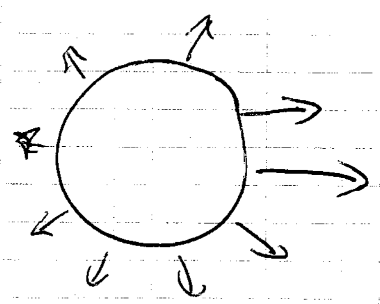
$$= -\frac{e/D}{2\pi\varepsilon_0}\cdot\ln\underbrace{\frac{r-\frac{r^2}{a'}}{a'-r}}_{\frac{r\cancel{(a'-r)}}{a'\cancel{(a'-r)}}} = \frac{e/D}{2\pi\varepsilon_0}\cdot\ln\frac{a'}{r}$$

$$= \frac{e/D}{2\pi\varepsilon_0}\cdot\ln\left(\frac{d}{2r}+\sqrt{\left(\frac{d}{2r}\right)^2-1}\right)$$

Zaradi simetrije: $\phi_D = -\phi_L$

$$U = \phi_L - \phi_D = 2\phi_L = \frac{e/D}{\pi\varepsilon_0}\cdot\ln\left(\frac{d}{2r}+\sqrt{\left(\frac{d}{2r}\right)^2-1}\right)$$

Tok:



$$I = \oint \vec{j}\,d\vec{S} = \int \sigma\cdot\vec{E}\,d\vec{S} =$$

$$= \frac{\sigma}{\varepsilon_0}\cdot\int\vec{D}\,d\vec{S} = \underline{\frac{\sigma}{\varepsilon_0}\cdot e}$$

$$R = \frac{U}{I} = \frac{1}{\pi\sigma D}\cdot\ln\left(\frac{d}{2r}+\sqrt{\left(\frac{d}{2r}\right)^2-1}\right)$$