1) Polje pri $\rho \to \infty$: $\vec{E} = E_0 \, e_x$

$$\underline{\varphi_0 = -E_0 \cdot \rho \cdot \cos\phi}$$

Polje okoli valja: ker ima znaje polje le komponento $\sim \cos\phi$, mora tudi okoli valja zadošča ista.

Znotraj: $\varphi^N = A_1 \cdot \rho \cdot \cos\phi$

Zunaj: $\varphi^Z = \underbrace{-E_0 \cdot \rho \cdot \cos\phi}_{\varphi_0} + \frac{B_1}{\rho} \cdot \cos\phi$ 0.25

Robni pogoji: i) $\varphi^N|_R = \varphi^Z|_R$

ii) $\varepsilon \cdot \frac{\partial \varphi^N}{\partial \rho}\Big|_R = \frac{\partial \varphi^Z}{\partial \rho}\Big|_R$ 0.25

i) $\to R A_1 = -E_0 R + \frac{B_1}{R}$

ii) $\to \varepsilon A_1 = -E_0 - \frac{B_1}{R^2}$

$$(\varepsilon + 1) A_1 = -2 E_0$$

$$A_1 = -\frac{2}{\varepsilon + 1} E_0$$

$$B_1 = R^2 (A_1 + E_0) = R^2 \frac{\varepsilon - 1}{\varepsilon + 1} E_0$$

Polje:

$$\varphi^N = -2\,\frac{E_0}{\varepsilon+1}\cdot \rho\cos\varphi = -2\,\frac{E_0}{\varepsilon+1}\cdot x$$

$$\underline{\underline{E^N = -\vec{\nabla}\varphi^N = \frac{2}{\varepsilon+1}\cdot E_0\cdot \hat{e}_x}} \qquad 0.25$$

$$\varphi^Z = -E_0\cdot\rho\cdot\cos\varphi + \frac{R^2}{\rho}\,\frac{\varepsilon-1}{\varepsilon+1}\,E_0\cdot\cos\varphi$$

0.25

$$E^Z = -\vec{\nabla}\varphi^Z = E_0\left(1+\frac{R^2}{\rho^2}\,\frac{\varepsilon-1}{\varepsilon+1}\right)\cos\varphi\cdot\hat{e}_r$$

$$+E_0\cdot\rho\left(-1+\frac{R^2}{\rho^2}\,\frac{\varepsilon-1}{\varepsilon+1}\right)\sin\varphi\cdot\hat{e}_\varphi$$

2) Nastavek: $\vec{E} = \vec{E}_0 \cdot e^{i(\vec{k}\vec{r} - \omega t)}$

Maxwell:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\downarrow$$

$$i\vec{k} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\rho = \varepsilon_0 \cdot i\vec{k} \cdot \vec{E}$$

$$\vec{\nabla}\rho = i\vec{k} \cdot \rho = -\varepsilon_0 \vec{k}(\vec{k}\vec{E})$$ 0.25

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \cdot \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\mu_0 \cdot \frac{\partial \vec{j}}{\partial t} = \vec{\nabla} \times \left(\frac{\partial \vec{B}}{\partial t}\right) - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$= -\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\frac{\partial \vec{j}}{\partial t} = \frac{1}{\mu_0}\left(\underbrace{\vec{k} \times (\vec{k} \times \vec{E})}_{\vec{k}(\vec{k}\cdot\vec{E}) - k^2 \cdot \vec{E}}\right) + \varepsilon_0 \cdot \omega^2 \cdot \vec{E}$$ 0.25

$$\vec{k}\cdot(\vec{k}\vec{E}) - k^2\vec{E} + \varepsilon_0\mu_0 \omega^2 \vec{E} - \varepsilon_0\mu_0 c^2 \vec{k}(\vec{k}\vec{E})$$

$$c^2\left(\vec{k}(\vec{k}\vec{E}) - k^2\vec{E}\right) + (\omega^2 - \omega_p^2)\underbrace{\vec{E}}_{= \varepsilon_0\mu_0\omega_p^2\vec{E}} - c^2\vec{k}(\vec{k}\vec{E}) = 0$$

b) $\underline{\vec{E} \parallel \vec{k}}$:

$$\underline{\omega^2 = \omega_p^2 + c^2 k^2}$$ 0.25

c) $\underline{\vec{E} \perp \vec{k}}$:

$$\underline{\omega^2 = \omega_p^2 + c^2 k^2}$$ 0.25

3) $E = 2 \cdot m_\pi c^2 = \gamma \cdot m_\pi c^2$

$\gamma = 2 \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad 1-\beta^2 = \frac{1}{\gamma^2}$

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{\frac{3}{4}}$$

a) $\vec{p}$ ϑ $\qquad p_\gamma = \frac{E_\gamma}{c}\begin{pmatrix} \cos\vartheta \\ \sin\vartheta \\ 0 \\ 1 \end{pmatrix}$

$$\vec{p}_\gamma = E_\gamma / c$$

L.T.: $\frac{E_\gamma'}{c} = -\beta\gamma \cdot p_x + \gamma \cdot \frac{E_\gamma}{c}$

$$\Rightarrow \underline{E_\gamma' = E_\gamma(-\beta \cdot \cos\vartheta + 1) \cdot \gamma}$$

$E_\gamma' = \frac{1}{2} m_\pi c^2$; $\quad E_\gamma' = E_\gamma$ če $\underbrace{\gamma(-\beta \cdot \cos\vartheta + 1)}_{=1}$

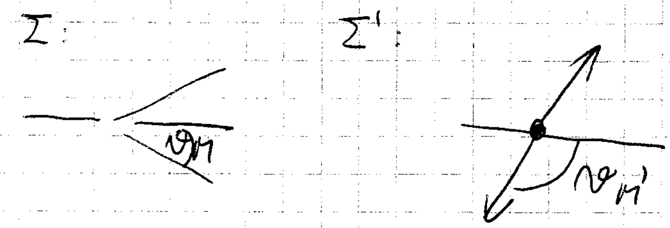
$$-\beta\cos\vartheta_M + 1 = \frac{1}{\gamma}$$

$$\cos\vartheta_M = \frac{1}{\beta}\left(1-\frac{1}{\gamma}\right) = \frac{2}{\sqrt{3}}\left(1-\frac{1}{2}\right) = \frac{1}{\sqrt{3}}$$

$$\vartheta_M = \arccos\frac{1}{\sqrt{3}}$$

$E > \frac{1}{2} m_\pi c^2$ velja, če je $\underline{0 \le \vartheta < \vartheta_M}$

b)



V Σ: $p_\gamma = \frac{E}{c}$ Σ': $p_\gamma' = \frac{E'}{c} = \frac{E}{c} = p_\gamma$

Hkrati velja $(p_\gamma)_y = (p_\gamma')_y$ (Lorentzova transformacij

$$p_\gamma \cdot \sin\vartheta_M = p_\gamma' \cdot \sin\vartheta_M'$$

$$\sin\vartheta_M = \sin\vartheta_M'$$

$$\underline{\underline{\vartheta_M' = \pi - \vartheta_M}} = \arccos\left(-\frac{1}{\sqrt{3}}\right)$$

Alternativno: izpeljemo izraz $\cos\vartheta' = \frac{\cos\vartheta - \beta}{1-\beta\cdot\cos\vartheta}$

Verjetnost: $$P_{E'>E} = P_{\vartheta'<\vartheta} = \frac{2\pi\int_0^{\vartheta_M'}\sin\vartheta\, d\vartheta}{2\pi\int_0^{\pi}\sin\vartheta\, d\vartheta}$$

$$= \frac{1-\cos\vartheta_M'}{2} = \frac{1+\frac{1}{\sqrt{3}}}{2} = \underline{\underline{79\%}}$$

4) Izračunamo iz

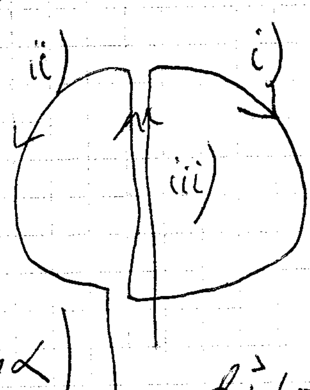
$$\vec{A} = \frac{\mu_0}{4\pi} \oint \frac{e^{-i\omega\left(t - \frac{|\vec{r}-\vec{r}'|}{c}\right)}}{|\vec{r}-\vec{r}'|} I_0 \, d\vec{r}' =$$

$$\simeq \frac{\mu_0}{4\pi r} \cdot I_0 \, e^{i(kr-\omega t)} \cdot \oint e^{-ik\frac{\vec{r}\vec{r}'}{r}} \, d\vec{r}'$$

Razvoj $$e^{-ik\hat{n}\vec{r}'} \approx 1 - \underbrace{ik\hat{n}\vec{r}'}_{\substack{\text{Dipolni člen} \\ =0}} - \underbrace{\tfrac{1}{2}k^2(\hat{n}\cdot\vec{r}')^2}_{\text{kvadrupolni}} + \ldots$$

$$\vec{A} = \frac{\mu_0}{4\pi r} \cdot I_0 \cdot e^{i(kr-\omega t)} \cdot \underbrace{\oint \left(-\tfrac{1}{2}k^2 (\hat{n}\cdot\vec{r}')^2\right) d\vec{r}'}_{= I}$$

I razdelimo na:



i) $$\vec{r}' = R \begin{pmatrix} \sin\alpha \\ \cos\alpha \\ 0 \end{pmatrix} \qquad d\vec{r}' = R \begin{pmatrix} \cos\alpha \\ -\sin\alpha \\ 0 \end{pmatrix} d\alpha$$

$$I_{i)} = -\frac{1}{2}k^2 \cdot \int_0^{\pi} R^3 \left(n_x \cdot \sin\alpha + n_y \cdot \cos\alpha\right)^2 \begin{pmatrix} \cos\alpha \\ -\sin\alpha \\ 0 \end{pmatrix}$$

Pomožni račun: $\int_0^\pi \sin^2\alpha \cos\alpha \, d\alpha = 0$

$$\int_0^\pi \sin\alpha \cdot \cos^2\alpha \, d\alpha = \int_{-1}^{1} \cos^2\alpha \, d(\cos\alpha) = \frac{2}{3}$$

$$\int_0^\pi \sin^3\alpha \, d\alpha = \int_{-1}^{1} (1-\cos^2\alpha) \, d(\cos\alpha) = \frac{4}{3}$$

$$I_{i)} = -\frac{1}{2} k^2 R^3 \begin{pmatrix} 2 m_x m_y \cdot \frac{2}{3} \\ -\frac{2}{3} m_y^2 - \frac{4}{3} m_x^2 \\ 0 \end{pmatrix}$$

$$I_{ii)} = -\frac{1}{2} k^2 \int_0^\pi R^3 \left(-m_x \sin\alpha + m_y \cdot \cos\alpha\right)^2 \begin{pmatrix} -\cos\alpha \\ -\sin\alpha \\ 0 \end{pmatrix} d\alpha$$

$$= I_{i)}$$

$$I_{iii)} = -\frac{1}{2} k^2 \cdot 2 \int_{-R}^{R} \cancel{y}'^2 m_y^2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} dy' = -\frac{1}{2} k^2 \cdot \frac{\cancel{4}}{3} R^3 m_y^2 \hat{e}_y$$

$$I = I_i + I_{ii} + I_{iii} = -\frac{1}{2} k^2 R^3 \frac{8}{3} \begin{pmatrix} m_x m_y \\ -m_x^2 \\ 0 \end{pmatrix}$$

$$\vec{A} = -\frac{\mu_0}{3\pi r} k^2 R^3 I_0 \cdot e^{i(kr-\omega t)} \cdot m_x \begin{pmatrix} m_y \\ -m_x \\ 0 \end{pmatrix}$$

Vidimo, da je $\underline{\vec{A} \perp \hat{n}}$

Zato je $\vec{S} = \frac{1}{2\mu_0} \cdot \vec{E}^* \times \vec{B}^* = \frac{c}{2\mu_0} |\vec{B}|^2 \cdot \hat{n} = \frac{c}{2\mu_0} \cdot |ik \times \vec{A}|^2 \cdot \hat{n}$

$$= \frac{ck^2}{2\mu_0} \cdot |\vec{A}|^2 \cdot \hat{n}$$

$$\vec{S} = \frac{c \cdot \mu_0}{18\pi^2 r^2} (kR)^6 \; I_0^2 \cdot n_x^2 \left(n_x^2 + n_y^2\right) \; \hat{n}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{I_0^2 \; (kR)^6}{18\pi^2 \; r^2} \cdot \sin^4\vartheta \cdot \cos^2\varphi \; \cdot \hat{n}$$

$$\frac{dP}{d\Omega} = r^2 \cdot \hat{n} \cdot \vec{S} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{I_0^2 \, (kR)^6}{18\pi^2} \sin^4\vartheta \cdot \cos^2\varphi$$