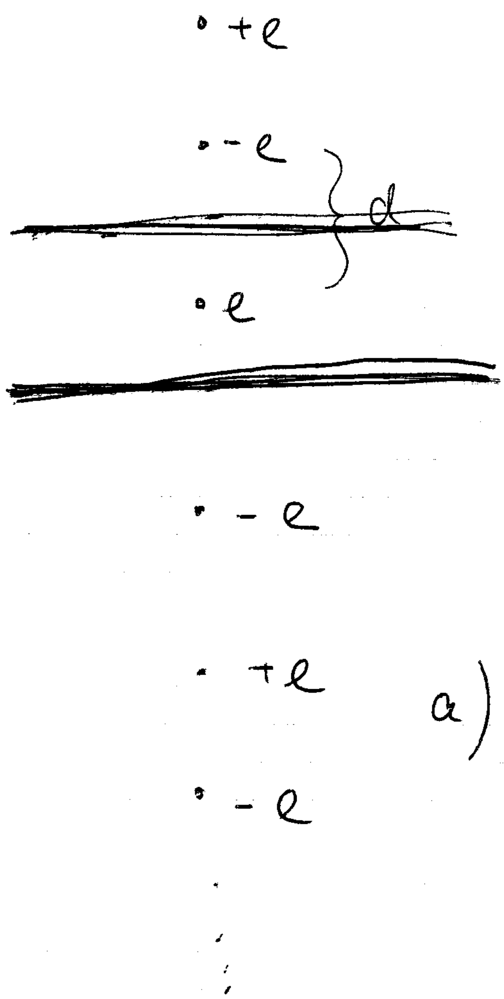
1) Da zadostimo obema robnima pogojema, moramo naboj ob x zrcaliti:



Potencial zrc. nabojev v središču:

$$\varphi = \frac{+e}{4\pi\varepsilon_0 d}\, 2\left(-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots\right)$$

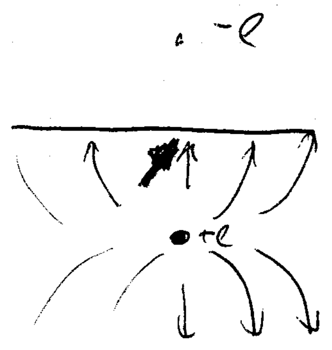
$$= -\frac{2e}{4\pi\varepsilon_0 d} \cdot \ln 2$$

a) Energija je $\frac{1}{2}\varphi \cdot e$, ker gre za zrc. slike samega naboja:

$$E = \frac{1}{2}\varphi \cdot e = -\frac{e^2}{4\pi\varepsilon_0 d} \cdot \ln 2$$

b) $$F = -\frac{\partial E}{\partial d} = -\frac{e^2}{4\pi\varepsilon_0 d^2} \cdot \ln 2$$ je privlačna.

c) Računamo polje v tej točki



$$\vec{E} = \hat{e}_z \cdot \left(\frac{e}{4\pi\varepsilon_0\left(\frac{d}{2}\right)^2} + \frac{e}{4\pi\varepsilon_0\left(\frac{d}{2}\right)^2} - \frac{e}{4\pi\varepsilon_0\left(\frac{3d}{2}\right)^2} - \frac{e}{4\pi\varepsilon_0\left(\frac{3d}{2}\right)^2} + \frac{2\cdot e}{4\pi\varepsilon_0\left(\frac{5d}{2}\right)^2} - \dots\right)$$

$$E_z = \frac{2e}{\pi \varepsilon_0 d^2}\left(1 - \frac{1}{3^2} + \frac{1}{5^2} - \dots\right) = \frac{2G \cdot e}{\pi \varepsilon_0 d^2}$$

$$\sigma = \varepsilon_0 \left(\underbrace{\cancel{E_z^{zunaj}}}_{=0} - E_z\right) = -\frac{2G \cdot e}{\pi\, d^2} \approx -0{,}58 \frac{e}{d^2}$$

2) $\phi(R, \vartheta) = U \cdot \cos\vartheta$

i) $\underline{r > R}$: $\phi = \sum_l \frac{B_l}{r^{l+1}} P_l(\cos\vartheta)$

r.p.: $\sum_l \frac{B_l}{R^{l+1}} P_l(\cos\vartheta) = U \cdot \overbrace{\cos\vartheta}^{= P_1(\cos\vartheta)}$

$B_l \neq 0$ edino pri $\underline{l = 1}$

$\frac{B_1}{R^2} = U \qquad B_1 = U \cdot R^2$

ii) $\underline{r < R}$: $\phi = \sum_l A_l \cdot r^l \cdot P_l(\cos\vartheta)$

$\sum_l A_l \cdot R^l \cdot P_l(\cos\vartheta) = U \cdot P_1(\cos\vartheta)$

$A_1 = \frac{U}{R}$, ostali $\emptyset$.

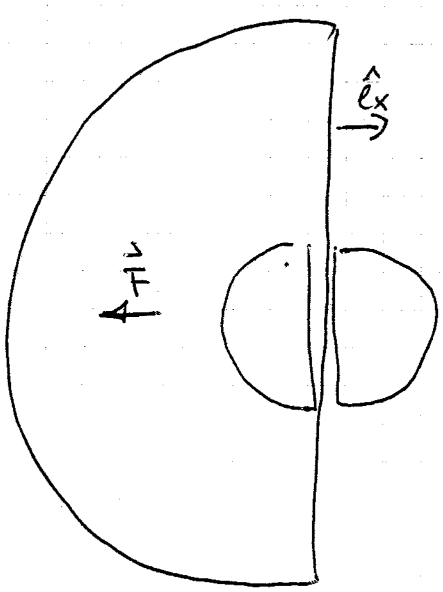
$$\phi(r,\vartheta) = \begin{cases} U R^2/r^2 & r>R \\ \frac{U}{R}\cdot r & r<R \end{cases} \cdot \cos\vartheta$$

b) $\vec{E} = -\vec{\nabla}\phi$

<u>r<R</u>: $\phi = \frac{Ur}{R}\cdot\cos\vartheta = \frac{Uz}{R}$

$$E = -\frac{U}{R}\cdot\hat{e}_z$$

<u>r>R</u>: $\vec{E} = -\hat{e}_r\cdot\frac{\partial\phi}{\partial r} - \frac{1}{r}\cdot\hat{e}_\vartheta\frac{\partial\phi}{\partial\vartheta} =$



$$= \frac{2UR^2}{r^3}\cos\vartheta\,\hat{e}_r + \frac{1}{r}\cdot\frac{UR^2}{r^2}\cdot\sin\vartheta\cdot\hat{e}_\vartheta$$

$$F_x = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dy\,dz\; T_{xx}$$

$$T_{xx} = \varepsilon_0\left(\underbrace{E_xE_x}_{=0 \text{ pri } x=0} - \frac{1}{2}\vec{E}^2\right) = -\frac{\varepsilon_0}{2}E^2$$

(!)

$$F_x = -\frac{\varepsilon_0}{2}\left(\int_0^R\int_0^{2\pi} r\,dr\,d\vartheta\; E^2_{\text{NOTR.}} + \int_R^\infty\int_0^{2\pi} r\,dr\,d\vartheta\, E^2_{\text{ZU}}\right)$$

$$= -\frac{\varepsilon_0}{2}\cdot\pi R^2\cdot\frac{U^2}{R^2} - \frac{\varepsilon_0}{2}\cdot\int_0^\infty\int_0^{2\pi} r\,dr\,d\vartheta\left(\frac{4U^2R^4}{r^6}\cos^2\vartheta + \frac{U^2R^4}{r^6}\sin^2\vartheta\right)$$

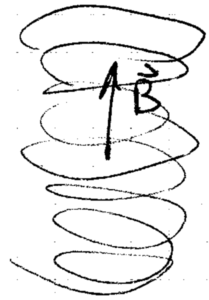
$$F_x = -\frac{\varepsilon_0}{2}\pi U^2 - \frac{\varepsilon_0}{2} U^2 R^4 \frac{1}{4R^4}\left(4\pi+\pi\right)$$

$$= -\frac{9}{8}\pi \varepsilon_0 U^2$$

sila je odbojna.

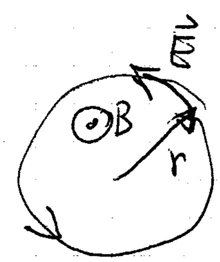
3) Polje u idealni tuljavi:

$$\vec B_0 = \frac{\mu_0 N I}{L}\cdot \hat e_z$$



a) Inducirano polje ima smjer $\hat e_\varphi$:

$$\oint \vec E\, d\vec l = -\oint \frac{\partial \vec B}{\partial t}\cdot d\vec S$$



$$E_\varphi 2\pi r = -\pi r^2 \cdot \frac{\partial B}{\partial t}$$

$$E_\varphi = -\frac{1}{2} r \cdot \frac{\partial B}{\partial t} = +\frac{1}{2} r \cdot \frac{B_0}{\tau}$$

b) $$\vec P = \frac{1}{\mu_0}\cdot \vec E\times\vec B = \frac{1}{\mu_0}\cdot\left(\frac{1}{2} r \frac{B_0}{\tau}\right)\cdot B_0\left(1-\frac{t}{\tau}\right)\cdot \underbrace{\hat e_\varphi \times \hat e_z}_{\hat e_r}$$

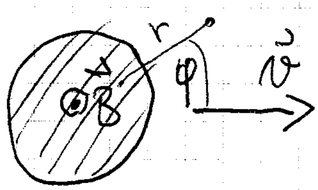
$$= \frac{1}{2\mu_0}\cdot r \cdot B_0^2 \frac{1}{\tau^2}(\tau - t)\cdot \hat e_r$$

Moč: $$P = \oint \vec{P}\, d\vec{S} = 2\pi R \cdot L \cdot P_r = \frac{1}{\mu_0}\pi R^2 L B_0^2 \frac{\tau - t}{\tau^2}$$

Energija: $$W = \int_0^{\tau} P\, d\tau = \frac{1}{\mu_0}\pi R^2 L \cdot B_0^2 \cdot \frac{\cancel{\tau^2}/2}{\cancel{\tau^2}} = \frac{1}{2\mu_0} \cdot \pi R^2 L \cdot B_0^2$$

Energija polja v valju: $$W_0 = \frac{1}{2\mu_0} \cdot B_0^2 \cdot V = \frac{1}{2\mu_0} B_0^2 \cdot \pi R^2 L$$ ✓

4)



a) Ker je sila na naboj v polju
$$F = e \cdot (\vec{E} + \vec{v} \times \vec{B})$$
sklepamo za tok:
$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B})$$

b) Ker gledamo ploščo v fiksnem sistemu, velja

$$\frac{\partial \rho}{\partial t}=0 \xrightarrow{\text{Kont. enačba}} \underline{\vec{\nabla}\cdot\vec{j}=0}$$

$$\vec{\nabla}\cdot(\vec{E}+\vec{v}\times\vec{B})=0$$

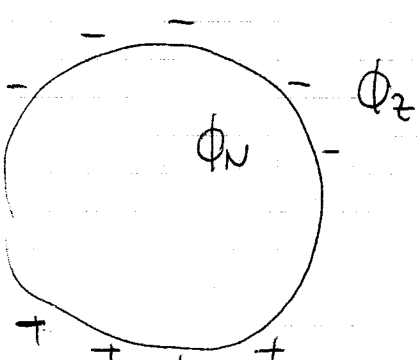
za $r<R$ in za $r>R$ je $\vec{B}$ konstanten,

torej $\underline{\vec{\nabla}(\vec{v}\times\vec{B})=0} \rightarrow \underline{\vec{\nabla}\cdot\vec{E}=0}$

Ker je v tem sistemu tudi $\frac{\partial \vec{B}}{\partial t}=0$, velja $\vec{\nabla}\times\vec{E}=0$,

$\underline{\vec{E}=-\vec{\nabla}\phi}$

Torej imamo za $r\neq R$ $\underline{\underline{\Delta\phi=0}}$



Pri $r=R$ pa se nabirejo naboji.

Vendar velja

$$\underline{\phi_N(R)=\phi_Z(R)}$$

Zaradi kontinuitete:

$$\vec{j}_N(R)\cdot\hat{e}_r=\vec{j}_Z(R)\cdot\hat{e}_r$$

$$\cancel{\sigma}\left(-\vec{\nabla}\phi_N + \underbrace{\vec{v}\times\vec{B}}_{-\hat{e}_y \cdot v \cdot B}\right)\hat{e}_r = \cancel{\sigma}\left(-\vec{\nabla}\phi_z\right)\hat{e}_r$$

$$-\frac{\partial \phi_N}{\partial R} - v \cdot B \cdot \sin\varphi = -\frac{\partial \phi_z}{\partial r} \Bigg|_{r=R}$$

c) Nastavek: $\phi_N(r) = C_1 \cdot r \cdot \sin\varphi$

$$\phi_z(r) = \frac{D_1}{r} \cdot \sin\varphi$$

Ostali členi so 0, ker nimajo prave kotne odvisnost.

R.P. 1: $C_1 \cdot R = \frac{D_1}{R} \Rightarrow D_1 = C_1 \cdot R^2$

R.P. 2: $-C_1 \cdot \cancel{\sin\varphi} - vB \sin\varphi = \frac{\overbrace{D_1}^{C_1}}{R^2} \cancel{\sin\varphi}$

$$C_1 = -\frac{vB}{2}$$

$$\phi_N = -\frac{vB}{2} \cdot r \sin\varphi$$

d) $\vec{j}_N = \sigma\cdot\left(-\vec{\nabla}\phi_N + \vec{v}\times\vec{B}\right) = \sigma\left(\frac{vB}{2}\hat{e}_y - vB\cdot\hat{e}_y\right) =$

$$= -\frac{vB}{2}\cdot\sigma\cdot\hat{e}_y$$

$$\vec{F} = \oint dV\; \vec{j}_N\times\vec{B} = \pi R^2\cdot d\cdot\sigma\frac{vB^2}{2}\left(\underbrace{-\hat{e}_y\times\hat{e}_z}_{-\hat{e}_x}\right)$$

$$= -\pi R^2 d\cdot\sigma\frac{B^2}{2}\vec{v}$$

Sila je linearno odvisna od hitrosti.